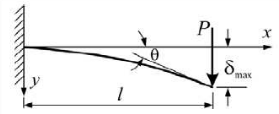
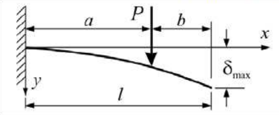
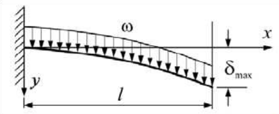
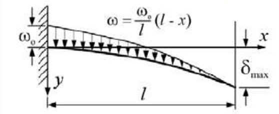
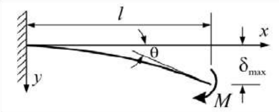
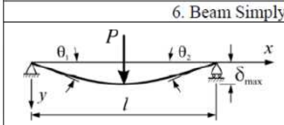
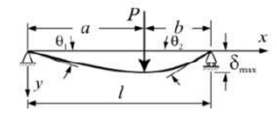
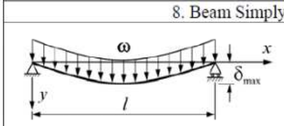
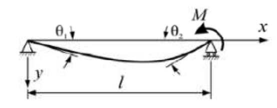
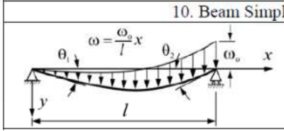
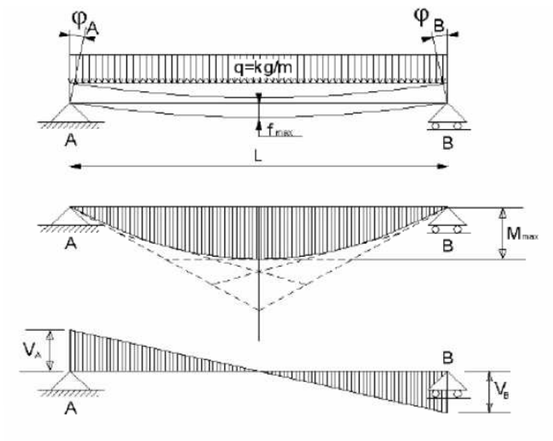
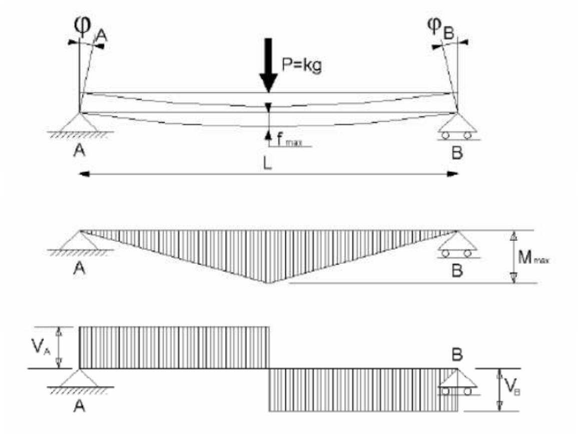
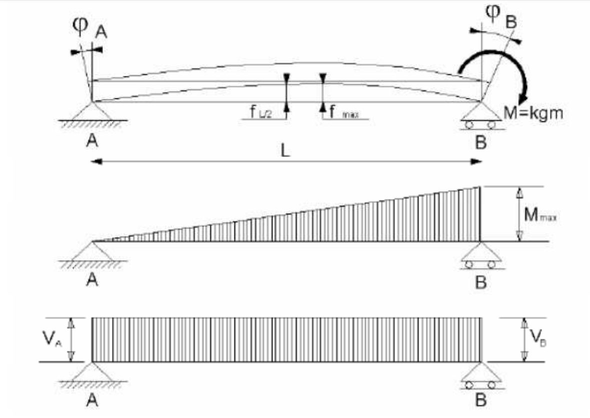
BEAM DEFLECTION FORMULAE

BEAM TYPE	SLOPE AT FREE END	DEFLECTION AT ANY SECTION IN TERMS OF x	MAXIMUM DEFLECTION
1. Cantilever Beam – Concentrated load P at the free end			
	$\theta = \dfrac{Pl^2}{2EI}$	$y = \dfrac{Px^2}{6EI}(3l - x)$	$\delta_{max} = \dfrac{Pl^3}{3EI}$
2. Cantilever Beam – Concentrated load P at any point			
	$\theta = \dfrac{Pa^2}{2EI}$	$y = \dfrac{Px^2}{6EI}(3a - x)$ for $0 < x < a$ $y = \dfrac{Pa^2}{6EI}(3x - a)$ for $a < x < l$	$\delta_{max} = \dfrac{Pa^2}{6EI}(3l - a)$
3. Cantilever Beam – Uniformly distributed load ω (N/m)			
	$\theta = \dfrac{\omega l^3}{6EI}$	$y = \dfrac{\omega x^2}{24EI}(x^2 + 6l^2 - 4lx)$	$\delta_{max} = \dfrac{\omega l^4}{8EI}$
4. Cantilever Beam – Uniformly varying load: Maximum intensity ω_o (N/m)			
	$\theta = \dfrac{\omega_o l^3}{24EI}$	$y = \dfrac{\omega_o x^2}{120lEI}(10l^3 - 10l^2x + 5lx^2 - x^3)$	$\delta_{max} = \dfrac{\omega_o l^4}{30EI}$
5. Cantilever Beam – Couple moment M at the free end			
	$\theta = \dfrac{Ml}{EI}$	$y = \dfrac{Mx^2}{2EI}$	$\delta_{max} = \dfrac{Ml^2}{2EI}$

BEAM DEFLECTION FORMULAS

BEAM TYPE	SLOPE AT ENDS	DEFLECTION AT ANY SECTION IN TERMS OF x	MAXIMUM AND CENTER DEFLECTION
6. Beam Simply Supported at Ends – Concentrated load P at the center			
	$\theta_1 = \theta_2 = \dfrac{Pl^2}{16EI}$	$y = \dfrac{Px}{12EI}\left(\dfrac{3l^2}{4} - x^2\right)$ for $0 < x < \dfrac{l}{2}$	$\delta_{max} = \dfrac{Pl^3}{48EI}$
7. Beam Simply Supported at Ends – Concentrated load P at any point			
	$\theta_1 = \dfrac{Pb(l^2 - b^2)}{6lEI}$ $\theta_2 = \dfrac{Pab(2l - b)}{6lEI}$	$y = \dfrac{Pbx}{6lEI}(l^2 - x^2 - b^2)$ for $0 < x < a$ $y = \dfrac{Pb}{6lEI}\left[\dfrac{l}{b}(x-a)^3 + (l^2 - b^2)x - x^3\right]$ for $a < x < l$	$\delta_{max} = \dfrac{Pb(l^2 - b^2)^{3/2}}{9\sqrt{3}\,lEI}$ at $x = \sqrt{(l^2 - b^2)/3}$ $\delta = \dfrac{Pb}{48EI}(3l^2 - 4b^2)$ at the center, if $a > b$
8. Beam Simply Supported at Ends – Uniformly distributed load ω (N/m)			
	$\theta_1 = \theta_2 = \dfrac{\omega l^3}{24EI}$	$y = \dfrac{\omega x}{24EI}(l^3 - 2lx^2 + x^3)$	$\delta_{max} = \dfrac{5\omega l^4}{384EI}$
9. Beam Simply Supported at Ends – Couple moment M at the right end			
	$\theta_1 = \dfrac{Ml}{6EI}$ $\theta_2 = \dfrac{Ml}{3EI}$	$y = \dfrac{Mlx}{6EI}\left(1 - \dfrac{x^2}{l^2}\right)$	$\delta_{max} = \dfrac{Ml^2}{9\sqrt{3}\,EI}$ at $x = \dfrac{l}{\sqrt{3}}$ $\delta = \dfrac{Ml^2}{16EI}$ at the center
10. Beam Simply Supported at Ends – Uniformly varying load: Maximum intensity ω_o (N/m)			
	$\theta_1 = \dfrac{7\omega_o l^3}{360EI}$ $\theta_2 = \dfrac{\omega_o l^3}{45EI}$	$y = \dfrac{\omega_o x}{360lEI}(7l^4 - 10l^2x^2 + 3x^4)$	$\delta_{max} = 0.00652\dfrac{\omega_o l^4}{EI}$ at $x = 0.519l$ $\delta = 0.00651\dfrac{\omega_o l^4}{EI}$ at the center

	**SCHEMA A1** Supported-supported beam with uniform load q $\varphi_A = \varphi_B = \frac{q \cdot L^3}{24 \cdot E \cdot J}$ $f_{\max} = \frac{5}{384} \cdot \frac{q \cdot L^4}{E \cdot J}$ $M_{\max} = \frac{q \cdot L^2}{8}$ $V_A = V_B = \frac{q \cdot L}{2}$
	**SCHEMA A2** Supported-supported beam with centered load $\varphi_A = \varphi_B = \frac{P \cdot L^2}{16 \cdot E \cdot J}$ $f_{\max} = \frac{8}{384} \cdot \frac{P \cdot L^3}{E \cdot J}$ $M_{\max} = \frac{P \cdot L}{4}$ $V_A = V_B = \frac{P}{2}$



SCHEMA A3

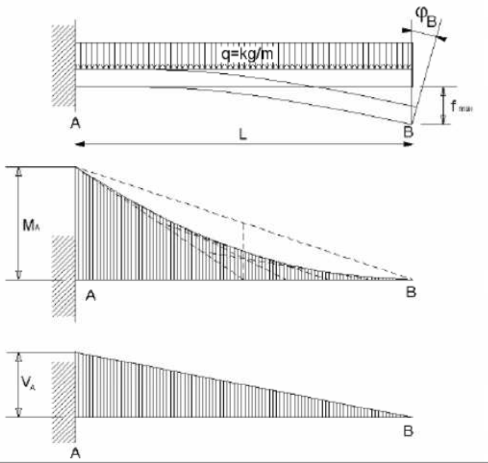
Supported-supported beam with moment in B

$$\varphi_A = \frac{M \cdot L}{6 \cdot E \cdot J} \qquad \varphi_B = \frac{M \cdot L}{3 \cdot E \cdot J}$$

$$f_{L/2} = \frac{1}{16} \cdot \frac{M \cdot L^2}{E \cdot J} \qquad f_{MAX} = 1{,}2064 \cdot f_{L/2}$$

$$M_A = 0 \qquad M_B = M$$

$$V_A = -V_B = \frac{M}{L}$$



SCHEMA B1

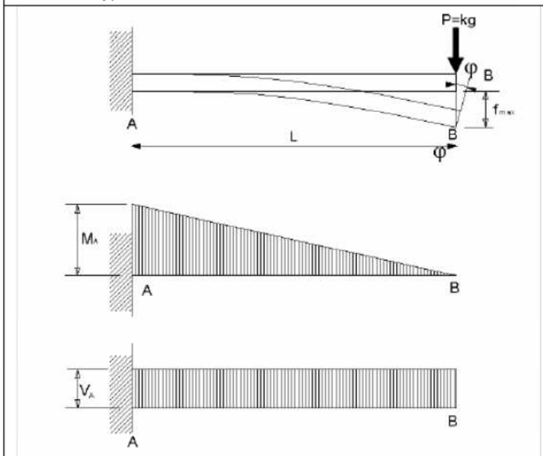
One end fixed beam with uniform load

$$\varphi_B = \frac{q \cdot L^3}{6 \cdot E \cdot J}$$

$$f_{\max} = \frac{1}{8} \cdot \frac{q \cdot L^4}{E \cdot J}$$

$$M_A = \frac{q \cdot L^2}{2}$$

$$V_A = q \cdot L$$



SCHEMA B2

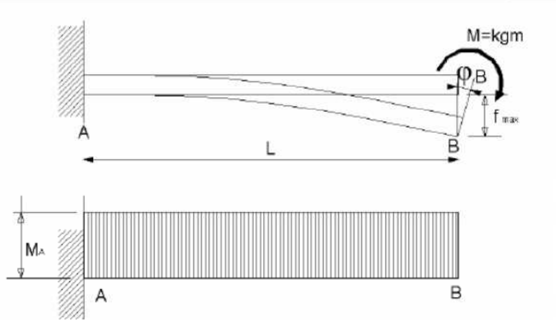
One end fixed beam with load in B

$$\varphi_B = \frac{P \cdot L^2}{2 \cdot E \cdot J}$$

$$f_B = \frac{1}{3} \cdot \frac{P \cdot L^3}{E \cdot J}$$

$$M_A = P \cdot L$$

$$V_A = -V_B = P$$



SCHEMA B3

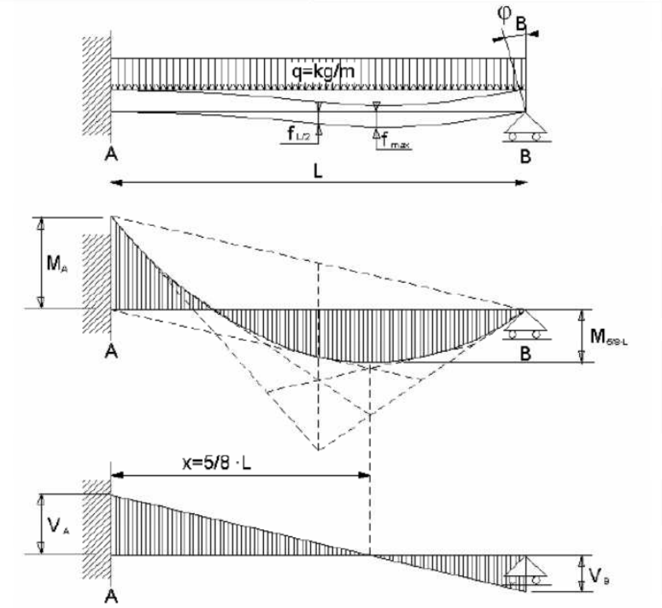
One end fixed beam with moment in B

$$\varphi_B = \frac{M \cdot L}{E \cdot J}$$

$$f_B = \frac{M \cdot L^2}{2 \cdot E \cdot J}$$

$$M_A = M \qquad V_A = 0$$

SCHEMA C1



Fixed-supported beam with uniform load q

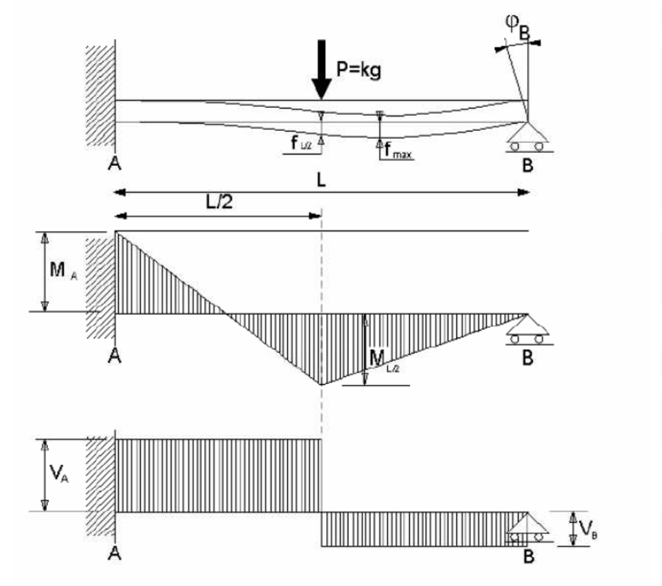
$$\varphi_B = \frac{q \cdot L^3}{48 \cdot E \cdot J}$$

$$f_{L/2} = \frac{2}{384} \cdot \frac{q \cdot L^4}{E \cdot J} \qquad f_{\max} = 1{,}04 \cdot f_{L/2}$$

$$M_A = \frac{q \cdot L^2}{8} \qquad M_{5/8 \cdot L} = \frac{q \cdot L^2}{14{,}22}$$

$$V_A = \frac{5}{8} \cdot q \cdot L \qquad V_B = \frac{3}{8} \cdot q \cdot L$$

SCHEMA C2



Fixed-supported beam with concentrated load in the center

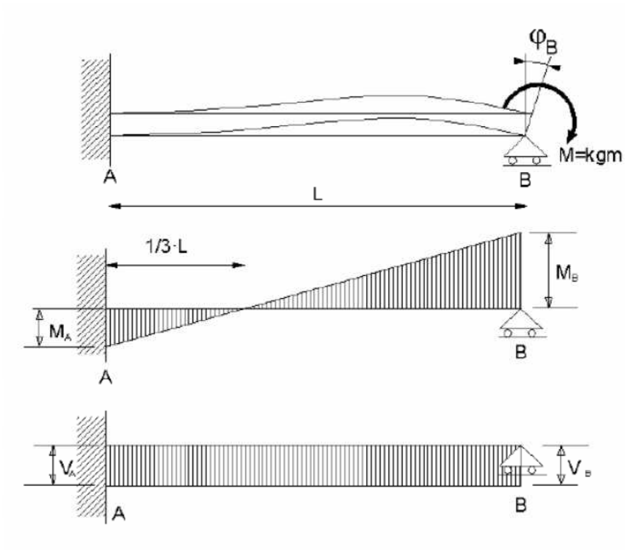
$$\varphi_B = \frac{P \cdot L^2}{32 \cdot E \cdot J}$$

$$f_{L/2} = \frac{3{,}5}{384} \cdot \frac{P \cdot L^3}{E \cdot J} \qquad f_{\max} = 1{,}022 \cdot f_{L/2}$$

$$M_A = \frac{3}{16} \cdot P \cdot L \qquad M_{L/2} = \frac{2{,}5}{16} \cdot P \cdot L$$

$$V_A = \frac{11}{16} \cdot P \qquad V_B = \frac{5}{16} \cdot P$$

SCHEMA C3



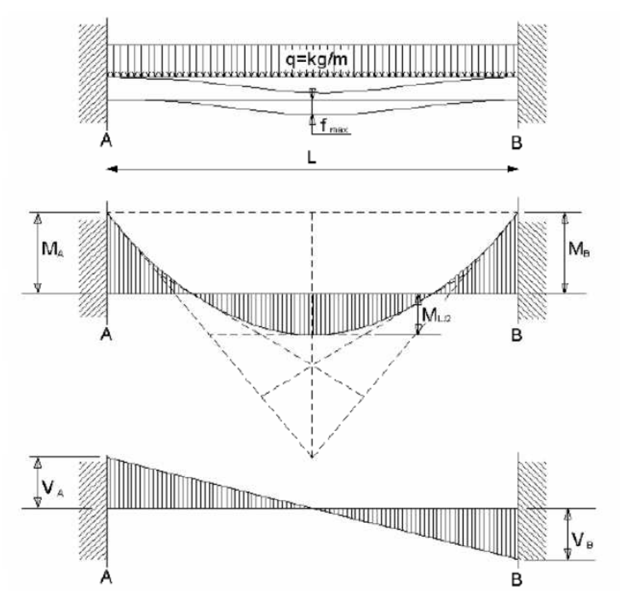
Fixed-supported beam with moment M in B

$$\varphi_B = \frac{M \cdot L}{4 \cdot E \cdot J}$$

$$M_A = \frac{M}{2} \qquad M_B = M$$

$$V_A = -V_B = \frac{3 \cdot M}{2 \cdot L}$$

SCHEMA D1



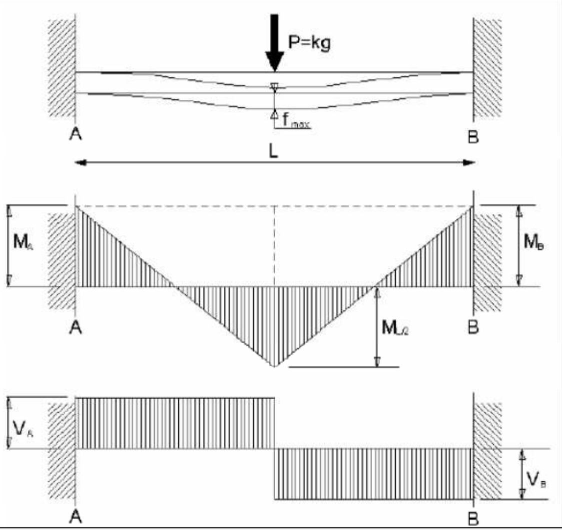
Fixed-fixed beam with uniform load

$$f_{MAX} = \frac{1}{384} \cdot \frac{q \cdot L^4}{E \cdot J}$$

$$M_A = M_B = \frac{q \cdot L^2}{12} \qquad M_{L/2} = \frac{q \cdot L^2}{24}$$

$$V_A = V_B = \frac{q \cdot L}{2}$$

SCHEMA D2



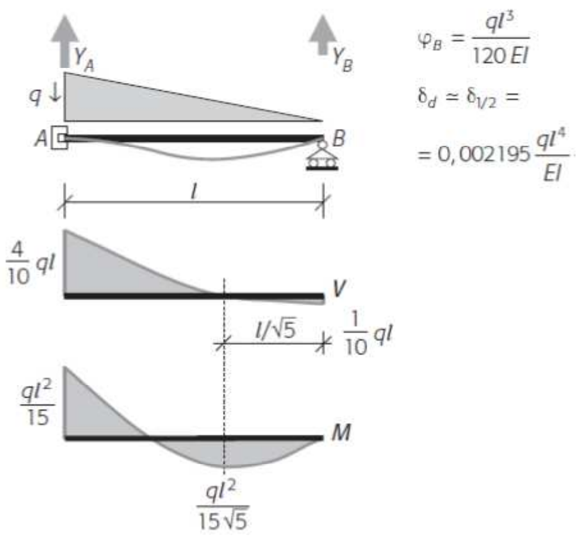
Fixed-fixed beam with concentrated load in the center

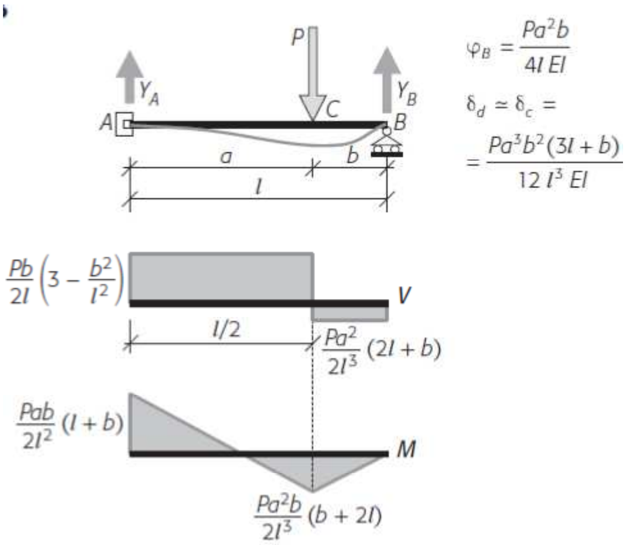
$$f_{Max} = \frac{2}{384} \cdot \frac{P \cdot L^3}{E \cdot J}$$

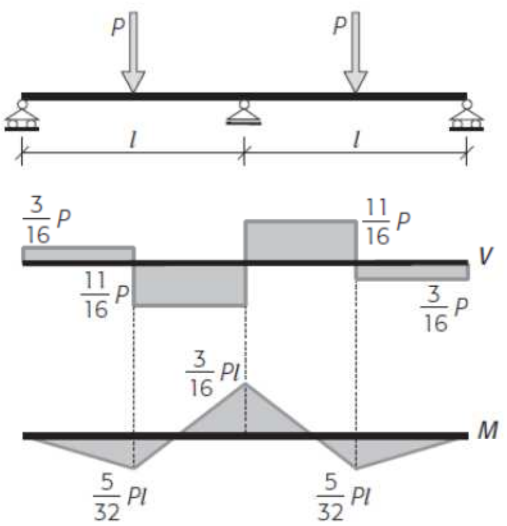
$$M_A = M_B = \frac{P \cdot L}{8} \qquad M_{L/2} = \frac{P \cdot L}{8}$$

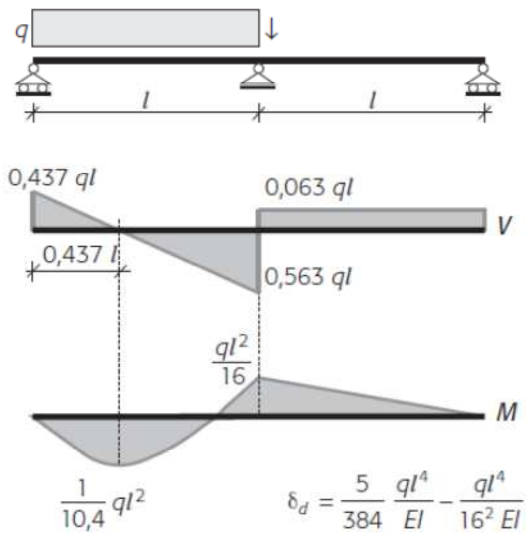
$$V_A = V_B = \frac{P}{2}$$

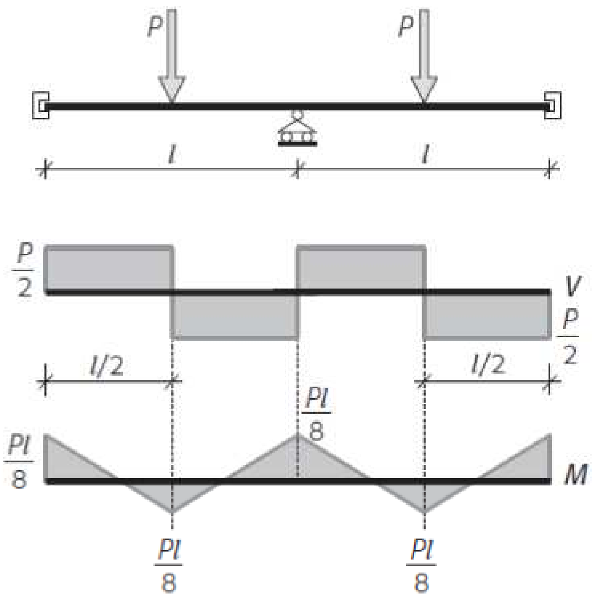
BEAM AND FRAME SCHEMES

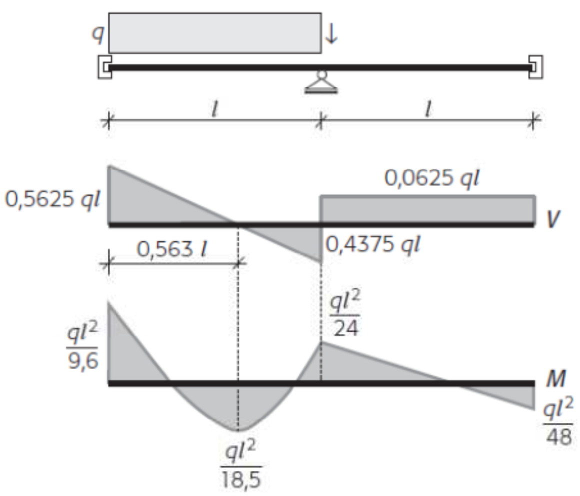


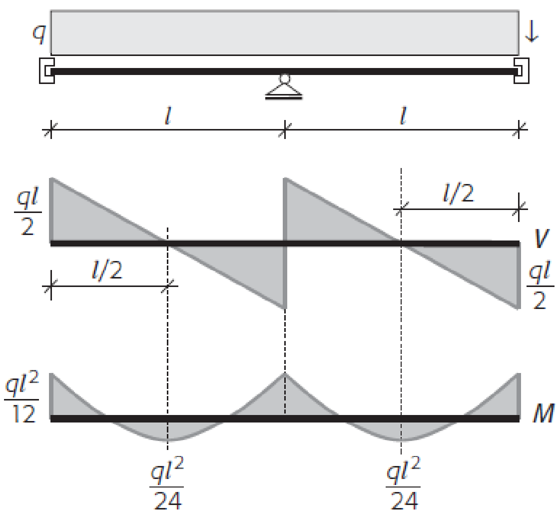


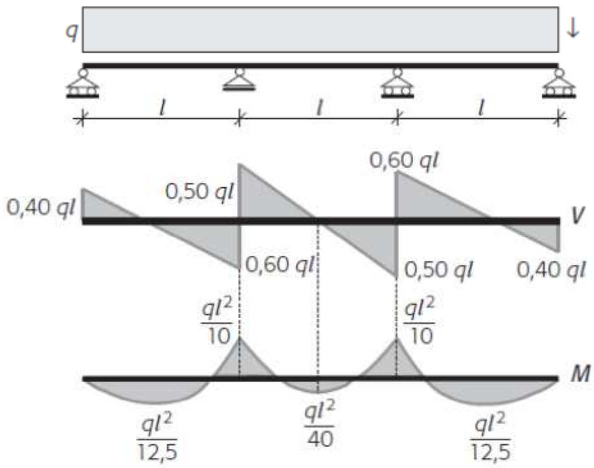


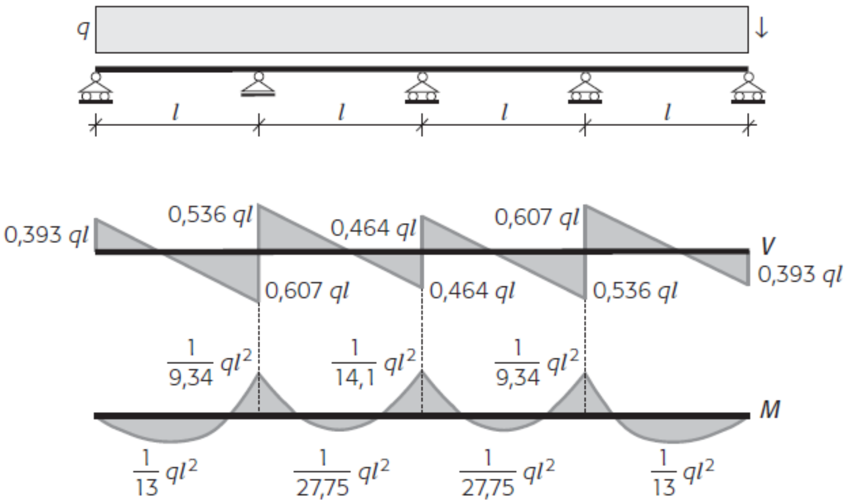


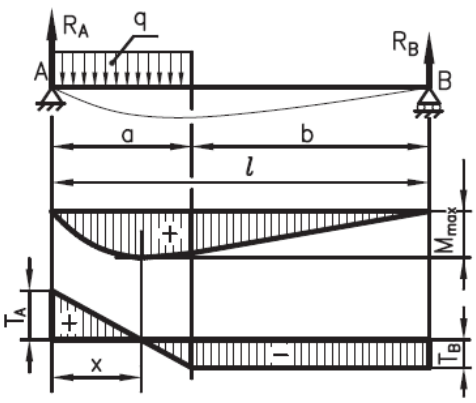












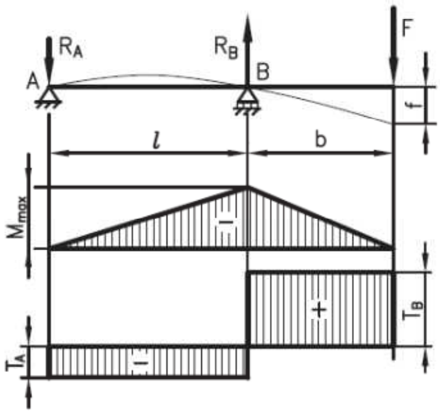
$$R_A = q \cdot a \cdot \frac{a+2b}{2l}\ ; \quad R_B = q \cdot \frac{a^2}{2l}$$

$$T_A = R_A; \quad T_B = -R_B$$

$$M_A = M_B = 0$$

$$M_{max} = \frac{R_A^2}{q}$$

$$x = \frac{R_A}{q}$$



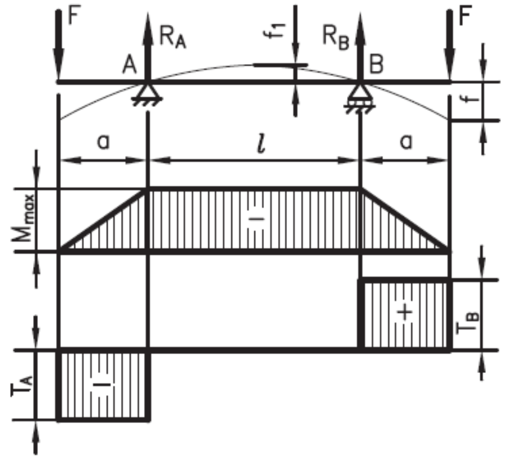
$$R_A = \frac{b}{l} \cdot F; \quad R_B = \frac{l+a}{l} \cdot F$$

$$T_A = -\frac{b}{l} \cdot F; \quad T_B = F$$

$$M_A = 0$$

$$M_{max} = -F \cdot b$$

$$f = \frac{F}{E \cdot I} \cdot \frac{(l+b) \cdot b^2}{3}$$



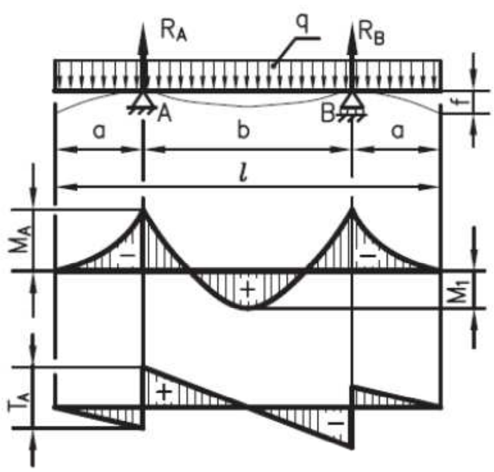
$$R_A = R_B = F$$

$$T_A = -R_A; \quad T_B = R_B$$

$$M_{max} = -F \cdot a$$

$$f_1 = \frac{F \cdot a \cdot l^2}{8 \cdot E \cdot I}$$

$$f = \frac{F \cdot a^2}{3 \cdot E \cdot I} \cdot \left(a + \frac{3l}{2}\right)$$



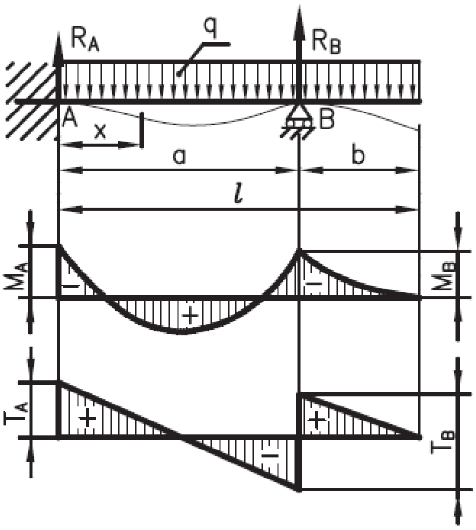
$$R_A = R_B = \frac{q \cdot l^2}{2}$$

$$T_A = |R_A|; \quad T_B = |R_B|$$

$$M_A = \frac{-q \cdot a}{2}$$

$$M_1 = \frac{q \cdot l^2}{4} \cdot \left(\frac{b}{l} - \frac{1}{2}\right)$$

$$f = \frac{q \cdot a}{24 \cdot E \cdot I} \cdot (3a^2 - b^3 + 6a^2 \cdot b)$$



$$R_A = \frac{q \cdot l}{2} \cdot \left(3 - \frac{3l}{2a} - \frac{a}{4l}\right)$$

$$R_B = \frac{q \cdot l}{2} \cdot \left(\frac{3l}{2a} + \frac{a}{4l} - 1\right)$$

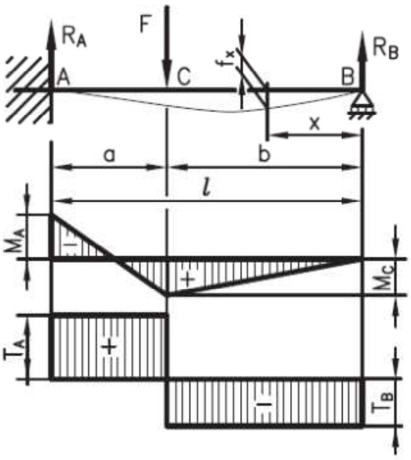
$$T_A = R_A; \quad T_B = |R_B|$$

$$M_A = q \cdot l \cdot \frac{2b^2 - a^2}{8l}$$

$$M_B = -\frac{q \cdot b^2}{2}$$

$$M_A = M_{max} \quad \text{se} \quad a > \sqrt{6} \cdot b$$

$$M_B = M_{max} \quad \text{se} \quad a < \sqrt{6} \cdot b$$



$$R_A = \frac{F}{2l^3} \cdot (3l^2 - b^2) \cdot b$$

$$R_B = \frac{F}{2l^3} \cdot (2l + b) \cdot a^2$$

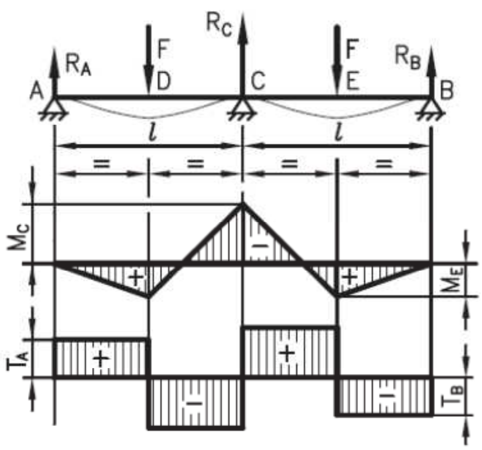
$$T_A = R_A; \quad T_B = -R_B$$

$$M_A = -\frac{F \cdot a \cdot b \cdot (l + b)}{2l^2}$$

$$M_C = \frac{F \cdot a^2 \cdot b \cdot (2l + b)}{2l^3}$$

$$f_C = \frac{F}{E \cdot I} \cdot \frac{a^3 \cdot b^2 \cdot (3l + b)}{12l^3}$$

$$f_x = \frac{F \cdot a^3}{12E \cdot I}\left[3b - (2l + b) \cdot \left(\frac{l - x}{l}\right)\right]\left(\frac{l - x}{l}\right)$$

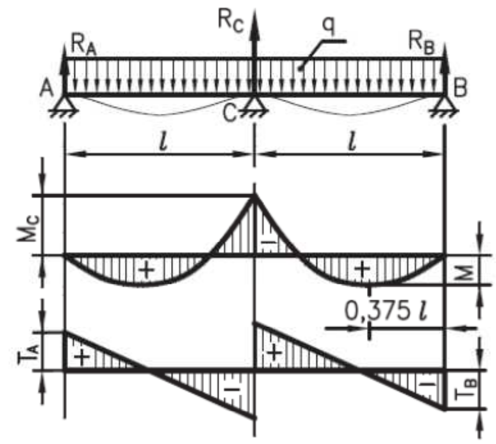


$$R_A = R_B = \frac{5}{16} \cdot F; \quad R_C = \frac{22}{16} \cdot F$$

$$T_A = R_A; \quad T_B = -R_B$$

$$M_A = M_B = \frac{5}{32} \cdot F \cdot l$$

$$M_C = -\frac{3}{16} \cdot F \cdot l$$

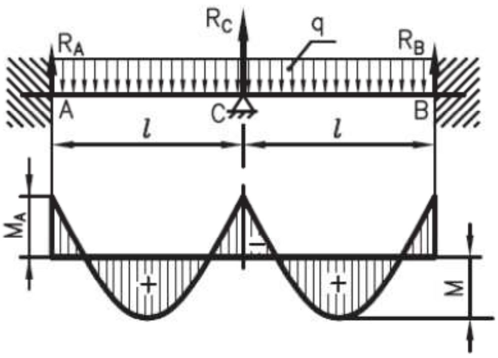


$$R_A = R_B = \frac{3}{8} \cdot q \cdot l; \quad R_C = \frac{5}{4} \cdot q \cdot l$$

$$T_A = R_A; \quad T_B = -R_B$$

$$M = \frac{9}{128} \cdot q \cdot l^2$$

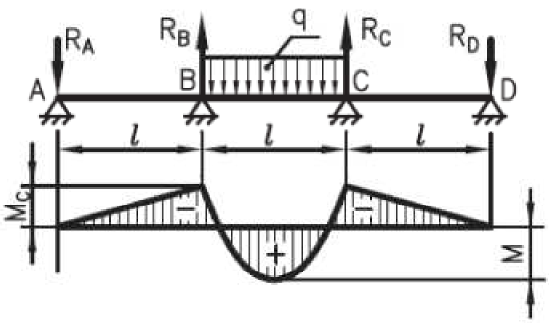
$$M_C = -\frac{1}{8} \cdot q \cdot l^2$$



$$R_A = R_B = \frac{1}{2} \cdot q \cdot l; \quad R_C = q \cdot l$$

$$M = \frac{1}{24} \cdot q \cdot l^2$$

$$M_A = M_B = M_C = -\frac{1}{12} \cdot q \cdot l^2$$

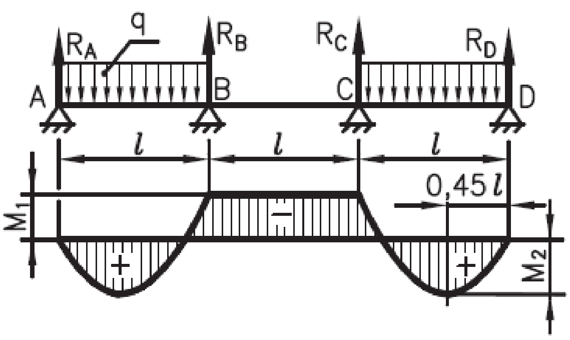


$$R_A = R_D = 0,005 \cdot q \cdot l$$

$$R_B = R_C = 0,550 \cdot q \cdot l$$

$$M = \frac{5}{67} \cdot q \cdot l^2$$

$$M_A = M_B = M_C = -\frac{1}{20} \cdot q \cdot l^2$$

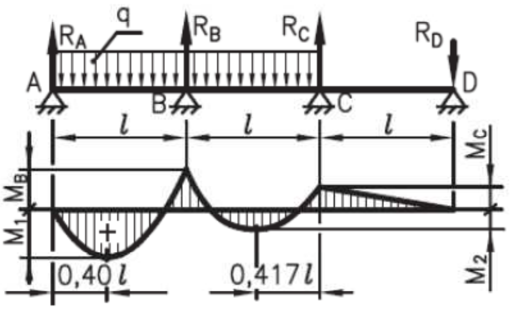


$$R_A = R_D = 0,450 \cdot q \cdot l$$

$$R_B = R_C = 0,550 \cdot q \cdot l$$

$$M_1 = -\frac{1}{20} \cdot q \cdot l^2$$

$$M_2 = \frac{10}{99} \cdot q \cdot l^2$$

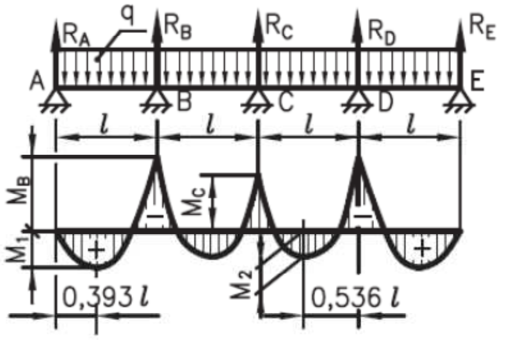


$$R_A = 0,383 \cdot q \cdot l; \quad R_B = 1,2 \cdot q \cdot l$$

$$R_C = 0,450 \cdot q \cdot l; \quad R_D = 0,033 \cdot q \cdot l$$

$$M_1 = \frac{1}{12,7} \cdot q \cdot l^2; \quad M_B = -\frac{1}{8,55} \cdot p \cdot l^2$$

$$M_C = -\frac{1}{30,3} \cdot q \cdot l^2; \quad M_2 = \frac{1}{18,3} \cdot q \cdot l^2$$



$$R_A = 0,393 \cdot q \cdot l \quad R_B = 1,143 \cdot q \cdot l$$

$$R_C = 0,929 \cdot q \cdot l; \quad R_D = 1,143 \cdot q \cdot l$$

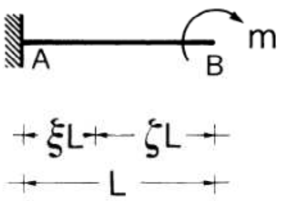
$$R_E = 0,393 \cdot q \cdot l$$

$$M_1 = \frac{1}{13} \cdot q \cdot l^2; \quad M_B = -\frac{1}{8,55} \cdot p \cdot l^2$$

$$M_C = -\frac{1}{14,1} \cdot q \cdot l^2; M_2 = \frac{1}{27,75} \cdot q \cdot l^2$$

Tabella 6. Travi ad asse rettilineo, dati per varie situazioni di carico e vincolo

1.



$Y_A = 0; \quad M_A = -m$

$T = cost = 0$

$M = cost = -m$

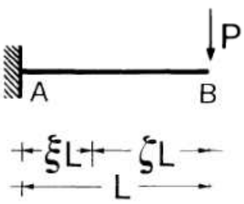
$\varphi = \frac{mL}{EI}\,\xi$

$\varphi_B = \frac{mL}{EI}$

$v = \frac{1}{2}\,\frac{mL^2}{EI}\,\xi^2$

$v_B = \frac{1}{2}\,\frac{mL^2}{EI}$

2.



$Y_A = P; \quad M_A = -PL$

$T = cost = P$

$M = -PL\,\zeta$

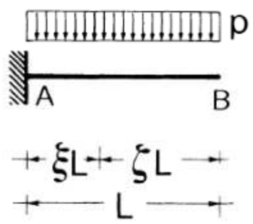
$\varphi = \frac{1}{2}\,\frac{PL^2}{EI}\,\xi\,(1+\zeta)$

$\varphi_B = \frac{1}{2}\,\frac{PL^2}{EI}$

$v = \frac{1}{6}\,\frac{PL^3}{EI}\,\xi^2\,(3-\xi)$

$v_B = \frac{1}{3}\,\frac{PL^3}{EI}$

3.



$Y_A = pL; \quad M_A = -\frac{1}{2}\,pL^2$

$T = pL\,\zeta$

$M = -\frac{1}{2}\,pL^2\,\zeta^2$

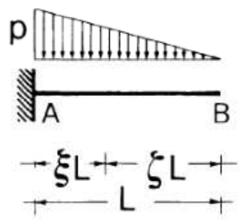
$\varphi = \frac{1}{6}\,\frac{pL^3}{EI}\,(1-\zeta^3)$

$\varphi_B = \frac{1}{6}\,\frac{pL^3}{EI}$

$v = \frac{1}{24}\,\frac{pL^4}{EI}\,[3-\zeta\,(4-\zeta^3)]$

$v_B = \frac{1}{8}\,\frac{pL^4}{EI}$

4.



$Y_A = \frac{1}{2}\,pL; \quad M_A = -\frac{1}{6}\,pL^2$

$T = \frac{1}{2}\,pL\,\zeta^2$

$M = -\frac{1}{6}\,pL^2\,\zeta^3$

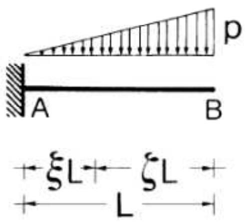
$\varphi = \frac{1}{24}\,\frac{pL^3}{EI}\,(1-\zeta^4)$

$\varphi_B = \frac{1}{24}\,\frac{pL^3}{EI}$

$v = \frac{1}{120}\,\frac{pL^4}{EI}\,[4-\zeta\,(5-\zeta^4)]$

$v_B = \frac{1}{30}\,\frac{pL^4}{EI}$

5.



$Y_A = \frac{1}{2}\,pL; \quad M_A = -\frac{1}{3}\,pL^2$

$T = \frac{1}{2}\,pL\,\zeta\,(2-\zeta)$

$M = -\frac{1}{6}\,pL^2\,(3-\zeta)\,\zeta^2$

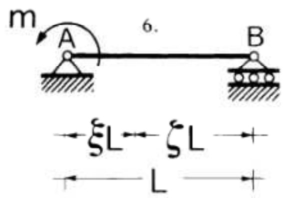
$\varphi = \frac{1}{24}\,\frac{pL^3}{EI}\,[3-\zeta^3\,(4-\zeta)]$

$\varphi_B = \frac{1}{8}\,\frac{pL^3}{EI}$

$v = \frac{1}{120}\,\frac{pL^4}{EI}\,[11-15\,\zeta+\zeta^4\,(5-\zeta)]$

$v_B = \frac{11}{120}\,\frac{pL^4}{EI}$

6.



$Y_A = -Y_B = \frac{m}{L}$

$T = cost = \frac{m}{L}$

$M = -m\,\zeta$

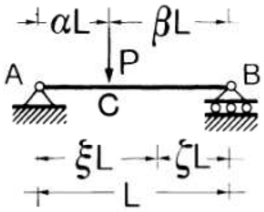
$\varphi = \frac{1}{6}\,\frac{mL}{EI}\,(1-3\,\zeta^2)$

$\varphi_A = -\frac{1}{3}\,\frac{mL}{EI}$

$\varphi_B = \frac{1}{6}\,\frac{mL}{EI}$

$v = -\frac{1}{6}\,\frac{mL^2}{EI}\,\zeta\,(1-\zeta^2)$

7.



– tronco AC:

$Y_A = P\,\beta$

$T = cost = P\beta$

$M = PL\,\beta\,\xi$

$\varphi = \frac{1}{6}\,\frac{PL^2}{EI}\,\beta\,(1-\beta^2-3\,\xi^2)$

$\varphi_A = \frac{1}{6}\,\frac{PL^2}{EI}\,\beta\,(1-\beta^2)$

$\varphi_C = \frac{1}{3}\,\frac{PL^2}{EI}\,\alpha\,\beta\,(\beta-\alpha)$

$v_A = \frac{1}{6}\,\frac{PL^3}{EI}\,\beta\,\xi\,(1-\beta^2-\xi^2)$

$v_C = \frac{1}{3}\,\frac{PL^3}{EI}\,\alpha^2\,\beta^2$

– tronco CB:

$Y_B = P\,\alpha$

$T = cost = -P\,\alpha$

$M = PL\,\alpha\,\zeta$

$M_{max} = M_C = PL\,\alpha\,\beta$

$\varphi = -\frac{1}{6}\,\frac{PL^2}{EI}\,\alpha\,(1-\alpha^2-3\,\zeta^2)$

$\varphi_B = -\frac{1}{6}\,\frac{PL^2}{EI}\,\alpha\,(1-\alpha^2)$

$v_B = \frac{1}{6}\,\frac{PL^3}{EI}\,\alpha\,\zeta\,(1-\alpha^2-\zeta^2)$

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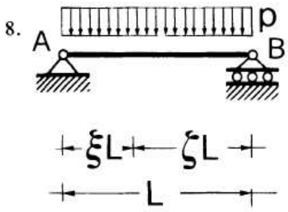
8.



$Y_A = Y_B = \frac{1}{2}\,pL$

$T = \frac{1}{2}\,pL\,(1-2\,\xi)$

$M = \frac{1}{2}\,pL^2\,\zeta\,\xi$

$\xi = \frac{1}{2}:\quad M = M_{\max} = \frac{1}{8}\,pL^2$

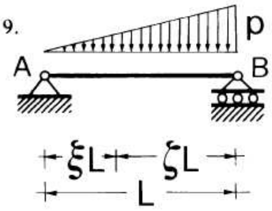
$\varphi = \frac{1}{24}\,\frac{pL^2}{EI}\,[1+2\,\xi^2\,(2\,\xi-3)]$

$\varphi_A = -\varphi_B = \frac{1}{24}\,\frac{pL^3}{EI}$

$v = \frac{1}{24}\,\frac{pL^4}{EI}\,\zeta\,\xi\,(1+\zeta\,\xi)$

$\xi = \frac{1}{2}:\quad v = v_{\max} = \frac{5}{384}\,\frac{PL^4}{EI}$

9.



$Y_A = \frac{1}{6}\,pL;\quad Y_B = \frac{1}{3}\,pL$

$T = \frac{1}{6}\,pL\,(1-3\,\xi^2)$

$M = \frac{1}{6}\,pL^2\,\zeta\,\xi\,(1+\xi)$

$\xi = \frac{\sqrt{3}}{3}:\quad M = M_{\max} = \frac{\sqrt{3}}{27}\,pL^2$

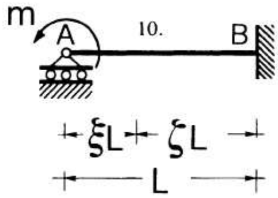
$\varphi = \frac{1}{360}\,\frac{pL^2}{EI}\,[7-15\,\xi^2\,(2-\xi^2)]$

$\varphi_A = \frac{7}{360}\,\frac{pL^3}{EI}$

$\varphi_B = -\frac{8}{360}\,\frac{pL^3}{EI}$

$v = \frac{1}{360}\,\frac{pL^4}{EI}\,\zeta\,\xi\,(1+\xi)\,(7-3\,\xi^2)$

10.



$Y_A = -Y_B = \frac{3}{2}\,\frac{m}{L}$

$M_B = \frac{1}{2}\,m$

$T = cost = \frac{3}{2}\,\frac{m}{L}$

$M = -\frac{1}{2}\,m\,(2-3\,\xi)$

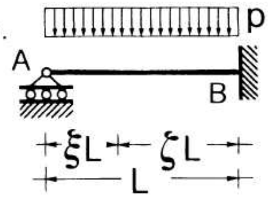
$\varphi = \frac{1}{4}\,\frac{mL}{EI}\,\zeta\,(2-3\,\zeta)$

$\varphi_A = -\frac{1}{4}\,\frac{mL}{EI}$

$v = -\frac{1}{4}\,\frac{mL^2}{EI}\,\xi\,\zeta^2$

$\xi = \frac{1}{3}:\quad v = v_{\min} = -\frac{1}{27}\,\frac{mL^2}{EI}$

11.



$Y_A = \frac{3}{8}\,pL;\quad Y_B = \frac{5}{8}\,pL$

$M_B = -\frac{1}{8}\,pL^2$

$T = pL\left(\frac{3}{8}-\xi\right)$

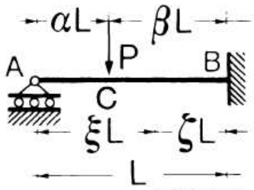
$M = \frac{1}{8}\,PL^2\,\xi\,(3-4\,\xi)$

$\varphi = \frac{1}{48}\,\frac{pL^3}{EI}\,\zeta\,(1+\xi-8\,\xi^2)$

$\varphi_A = \frac{1}{48}\,\frac{pL^3}{EI}$

$v = \frac{1}{48}\,\frac{pL^4}{EI}\,\xi\,\zeta^2\,(1+2\,\xi)$

12.



– tronco AC:

$Y_A = \frac{1}{2}\,P\,\beta^2\,(2+\alpha)$

$T = cost = Y_A$

$M = \frac{1}{2}\,PL\,\xi\,\beta^2\,(2+\alpha)$

– tronco CB:

$Y_B = \frac{1}{2}\,P\,\alpha\,(3-\alpha^2)$

$M_B = -\frac{1}{2}\,PL\,\alpha\,(1-\alpha^2)$

$T = cost = -Y_B$

$M = \frac{1}{2}\,PL\,\alpha\,(3\,\zeta+\alpha^2\,\xi-1)$

$M_C = PL\,\alpha\,\beta^2\,(2+\alpha)/2$

AC: $\varphi = \frac{1}{4}\,\frac{PL^2}{EI}\,\beta^2\,[\alpha-(2+\alpha)\,\xi^2];$

CB: $\varphi = \frac{1}{4}\,\frac{PL^2}{EI}\,\alpha\zeta[\alpha^2(2-\zeta)+3\zeta-2]$

$\varphi_A = \frac{1}{4}\,\frac{PL^2}{EI}\,\alpha\,\beta^2$

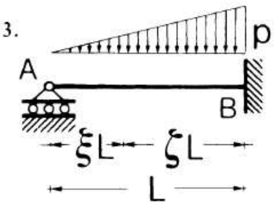
$\varphi_C = \frac{1}{4}\,\frac{PL^2}{EI}\,\alpha\,\beta^2\,(1-2\alpha-\alpha^2)$

AC: $v = \frac{1}{12}\,\frac{PL^3}{EI}\,\xi\beta^2\,[3\alpha-(2+\alpha)\,\xi^2]$

CB: $v = \frac{1}{12}\,\frac{PL^3}{EI}\,\alpha\zeta^2\,[3(1-\alpha^2)+ -(3-\alpha^2)\zeta]$

$v_C = \frac{1}{12}\,\frac{PL^3}{EI}\,\alpha^2\,\beta^3\,(3+\alpha)$

13.



$Y_A = \frac{1}{10}\,pL;\quad Y_B = \frac{2}{5}\,pL$

$M_B = -\frac{1}{15}\,pL^2$

$T = \frac{1}{10}\,pL\,(1-5\,\xi^2)$

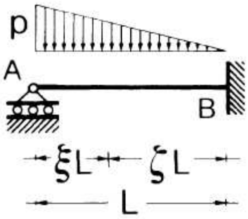
$M = \frac{1}{10}\,pL^2\,\xi\,(3-5\,\xi^2)$

$\varphi = \frac{1}{120}\,\frac{pL^3}{EI}\cdot\zeta\,(1+\xi)\,(1-5\xi^2)$

$\varphi_A = \frac{1}{120}\,\frac{pL^3}{EI}$

$v = \frac{1}{120}\,\frac{pL^4}{EI}\,\xi\,\zeta^2\,(1+\xi)^2$

14.



$Y_A = \frac{11}{40}\,pL;\quad Y_B = \frac{9}{40}\,pL$

$M_B = -\frac{7}{120}\,pL^2$

$T = \frac{1}{40}\,pL\,(20\,\zeta^2-9)$

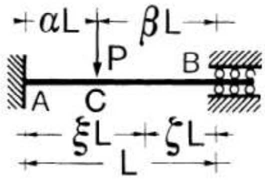
$M = \frac{1}{120}\,pL^2\,\xi\,(20\zeta^2+20\zeta-7)$

$\varphi = \frac{1}{240}\,\frac{pL^3}{EI}\,\zeta\,(27\,\zeta+ -10\,\zeta^3-14)$

$\varphi_A = \frac{1}{80}\,\frac{pL^3}{EI}$

$v = \frac{1}{240}\,\frac{pL^4}{EI}\,\xi\zeta^2\,(7-2\,\zeta-2\zeta^2)$

15.



– tronco AC:

$Y_A = P\,\beta^2\,(1+2\,\alpha)$

$M_A = -\,PL\,\alpha\,\beta^2$

$T = cost = Y_A$

$M = PL\,\beta^2\,[\xi\,(1+2\,\alpha)-\alpha]$

– tronco CB:

$Y_B = P\,\alpha^2\,(1+2\,\beta)$

$M_B = -\,PL\,\alpha^2\,\beta$

$T = cost = -\,Y_B$

$M = PL\,\alpha^2\,[\zeta\,(1+2\,\beta)-\beta]$

$M_C = 2\,PL\,\alpha^2\,\beta^2$

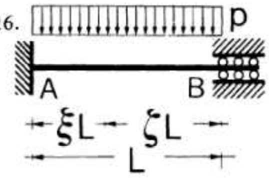
$\varphi = \frac{1}{2}\,\frac{PL^2}{EI}\,\beta^2\xi\,[2\alpha-\xi\,(1+2\alpha)]$ $\quad\varphi = -\,\frac{1}{2}\,\frac{PL^2}{EI}\,\alpha^2\zeta\,[2\beta-\zeta\,(1+2\beta)]$

$\varphi_c = \frac{1}{2}\,\frac{PL^2}{EI}\,\alpha^2\,\beta^2\,(1-2\,\alpha)$

$v = \frac{1}{6}\,\frac{PL^3}{EI}\,\beta^2\xi^2\,[3\alpha-\xi\,(1+2\alpha)]$ $\quad v = \frac{1}{6}\,\frac{PL^3}{EI}\,\alpha^2\zeta^2\,[3\beta-\zeta\,(1+2\beta)]$

$v_C = \frac{1}{3}\,\frac{PL^3}{EI}\,\alpha^3\,\beta^3$

16.



$Y_A = Y_B = \frac{1}{2}\,pL$

$M_A = M_B = -\,\frac{1}{12}\,pL^2$

$T = \frac{1}{2}\,pL\,(1-2\,\xi)$

$M = -\,\frac{1}{12}\,pL^2\,(1-6\,\xi\,\zeta)$

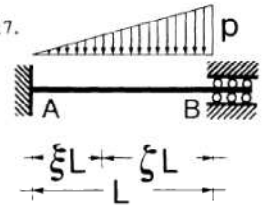
$\xi = \frac{1}{2}:\quad M = \frac{1}{24}\,pL^2$

$\varphi = \frac{1}{12}\,\frac{pL^2}{EI}\,\xi\,\zeta\,(1-2\,\xi)$

$v = \frac{1}{24}\,\frac{pL^4}{EI}\,\xi^2\,\zeta^2$

$\xi = \frac{1}{2}:\quad v = \frac{1}{384}\,\frac{pL^4}{EI}$

17.



$Y_A = \frac{3}{20}\,pL;\quad Y_B = \frac{7}{20}\,pL$

$M_A = -\,\frac{1}{30}\,pL^2;\quad M_B = \frac{1}{20}\,pL^2$

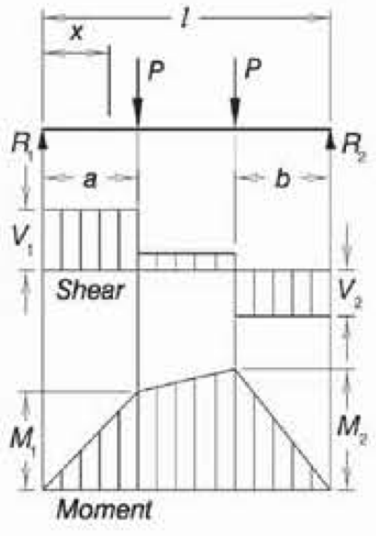
$T = \frac{1}{20}\,pL\,(3-10\,\xi^2)$

$M = -\,\frac{1}{60}\,pL^2\,(2-9\,\xi+10\,\xi^3)$

$\varphi = \frac{1}{120}\,\frac{pL^3}{EI}\,\xi\zeta\,(4-5\,\xi-5\,\xi^2)$

$v = \frac{1}{120}\,\frac{pL^4}{EI}\,\xi^2\,\zeta^2\,(2+\xi)$

10. SIMPLE BEAM — TWO EQUAL CONCENTRATED LOADS UNSYMMETRICALLY PLACED



$R_1 = V_1$ ($= V_{max}$ when $a < b$) .... $= \frac{P}{l}(l-a+b)$

$R_2 = V_2$ ($= V_{max}$ when $a > b$) .... $= \frac{P}{l}(l-b+a)$

V_x (when $a < x < (l-b)$) .... $= \frac{P}{l}(b-a)$

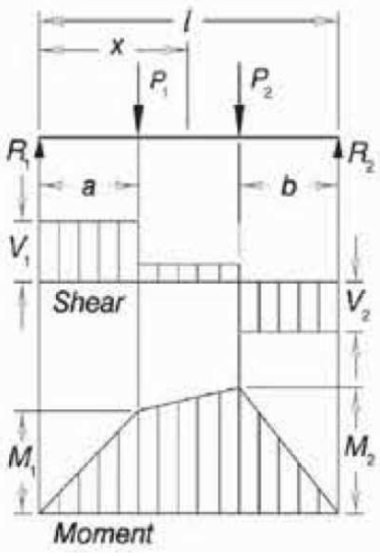
M_1 ($= M_{max}$ when $a > b$) .... $= R_1 a$

M_2 ($= M_{max}$ when $a < b$) .... $= R_2 b$

M_x (when $x < a$) .... $= R_1 x$

M_x (when $a < x < (l-b)$) .... $= R_1 x - P(x-a)$

11. SIMPLE BEAM — TWO UNEQUAL CONCENTRATED LOADS UNSYMMETRICALLY PLACED



$R_1 = V_1$ .... $= \frac{P_1(l-a)+P_2 b}{l}$

$R_2 = V_2$ .... $= \frac{P_1 a + P_2 (l-b)}{l}$

V_x (when $a < x < (l-b)$) .... $= R_1 - P_1$

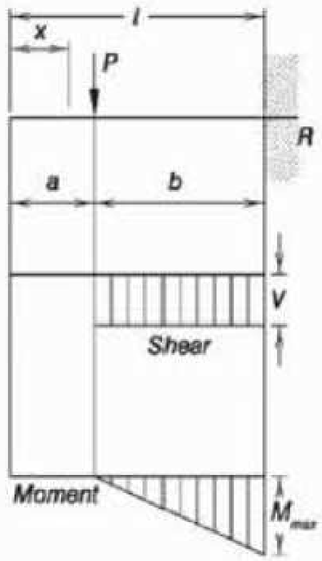
M_1 ($= M_{max}$ when $R_1 < P_1$) .... $= R_1 a$

M_2 ($= M_{max}$ when $R_2 < P_2$) .... $= R_2 b$

M_x (when $x < a$) .... $= R_1 x$

M_x (when $a < x < (l-b)$) .... $= R_1 x - P_1(x-a)$

21. CANTILEVERED BEAM — CONCENTRATED LOAD AT ANY POINT



Total Equiv. Uniform Load .... $= \frac{8Pb}{l}$

$R = V$ .... $= P$

M_{max} (at fixed end) .... $= Pb$

M_x (when $x > a$) .... $= P(x-a)$

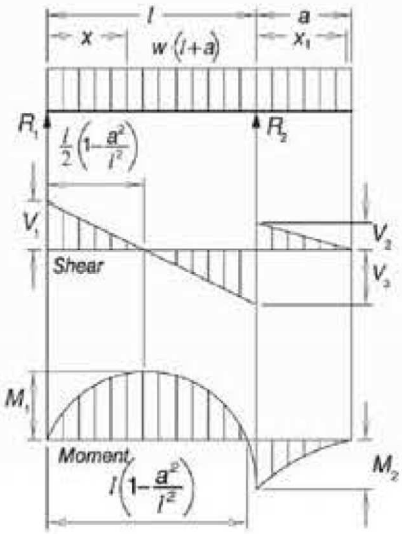
Δ_{max} (at free end) .... $= \frac{Pb^2}{6EI}(3l-b)$

Δ_a (at point of load) .... $= \frac{Pb^3}{3EI}$

Δ_x (when $x < a$) .... $= \frac{Pb^2}{6EI}(3l-3x-b)$

Δ_x (when $x > a$) .... $= \frac{P(l-x)^2}{6EI}(3b-l+x)$

24. BEAM OVERHANGING ONE SUPPORT — UNIFORMLY DISTRIBUTED LOAD



$R_1 = V_1$ .... $= \frac{w}{2l}\left(l^2 - a^2\right)$

$R_2 = V_2 + V_3$ .... $= \frac{w}{2l}(l+a)^2$

V_2 .... $= wa$

V_3 .... $= \frac{w}{2l}\left(l^2 + a^2\right)$

V_x (between supports) .... $= R_1 - wx$

V_{x_1} (for overhang) .... $= w(a - x_1)$

M_1 $\left(\text{at } x = \frac{l}{2}\left[1 - \frac{a^2}{l^2}\right]\right)$ .... $= \frac{w}{8l^2}(l+a)^2(l-a)^2$

M_2 (at R_2) .... $= \frac{wa^2}{2}$

M_x (between supports) .... $= \frac{wx}{2l}\left(l^2 - a^2 - xl\right)$

M_{x_1} (for overhang) .... $= \frac{w}{2}(a - x_1)^2$

Δ_x (between supports) .... $= \frac{wx}{24EIl}\left(l^4 - 2l^2x^2 + lx^3 - 2a^2l^2 + 2a^2x^2\right)$

Δ_{x_1} (for overhang) .... $= \frac{wx_1}{24EI}\left(4a^2l - l^3 + 6a^2x_1 - 4ax_1^2 + x_1^3\right)$

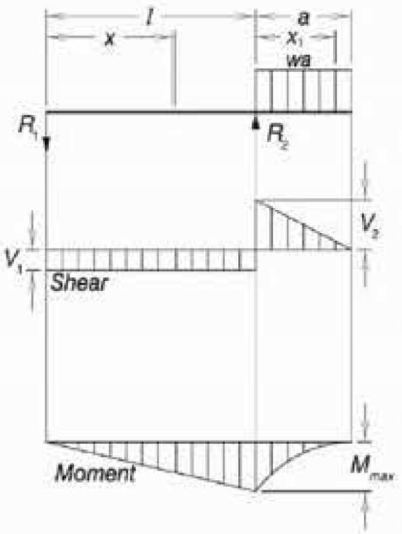
NOTE: For a negative value of Δ_x, deflection is upward.

25. BEAM OVERHANGING ONE SUPPORT — UNIFORMLY DISTRIBUTED LOAD ON OVERHANG

$R_1 = V_1$ .... $= \frac{wa^2}{2l}$

$R_2 = V_1 + V_2$ .... $= \frac{wa}{2l}(2l + a)$

V_2 .... $= wa$



V_{x_1} (for overhang) .... $= w(a - x_1)$

M_{max} (at R_2) .... $= \frac{wa^2}{2}$

M_x (between supports) .... $= \frac{wa^2x}{2l}$

M_{x_1} (for overhang) .... $= \frac{w}{2}(a - x_1)^2$

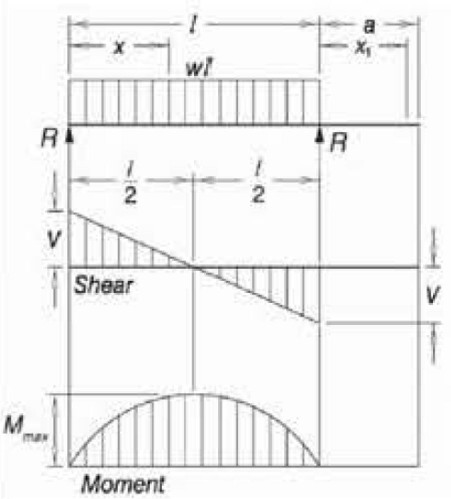
Δ_{max} $\left(\text{between supports at } x = \frac{l}{\sqrt{3}}\right) = \frac{wa^2l^2}{18\sqrt{3}EI} = 0.0321\frac{wa^2l^2}{EI}$

Δ_{max} (for overhang at $x_1 = a$)..... $= \frac{wa^3}{24EI}(4l + 3a)$

Δ_x (between supports) .... $= \frac{wa^2x}{12EIl}\left(l^2 - x^2\right)$

Δ_{x_1} (for overhang) .... $= \frac{wx_1}{24EI}\left(4a^2l + 6a^2x_1 - 4ax_1^2 + x_1^3\right)$

27. BEAM OVERHANGING ONE SUPPORT — UNIFORMLY DISTRIBUTED LOAD BETWEEN SUPPORTS



Total Equiv. Uniform Load .... $= wl$

$R = V$ .... $= \frac{wl}{2}$

V_x .... $= w\left(\frac{l}{2} - x\right)$

M_{max} (at center) .... $= \frac{wl^2}{8}$

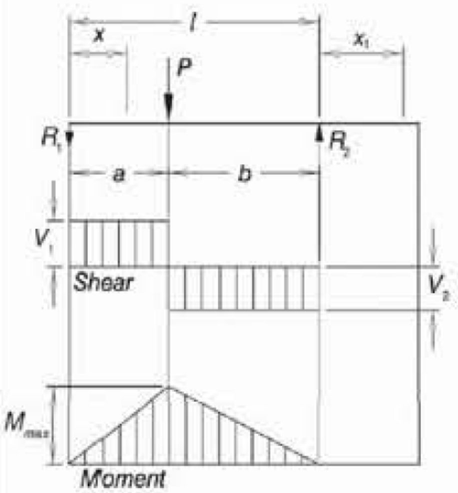
M_x .... $= \frac{wx}{2}(l - x)$

Δ_{max} (at center) .... $= \frac{5wl^4}{384EI}$

Δ_x .... $= \frac{wx}{24EIl}\left(l^3 - 2lx^2 + x^3\right)$

Δ_{x_1} .... $= \frac{wl^3 x_1}{24EI}$

28. BEAM OVERHANGING ONE SUPPORT — CONCENTRATED LOAD AT ANY POINT BETWEEN SUPPORTS



Total Equiv. Uniform Load .... $= \frac{8Pab}{l^2}$

$R_1 = V_1$ ($= V_{max}$ when $a < b$) .... $= \frac{Pb}{l}$

$R_2 = V_2$ ($= V_{max}$ when $a > b$) .... $= \frac{Pa}{l}$

M_{max} (at point of load) .... $= \frac{Pab}{l}$

M_x (when $x < a$) .... $= \frac{Pbx}{l}$

Δ_{max} $\left(\text{at } x = \sqrt{\frac{a(a+2b)}{3}} \text{ when } a > b\right)$ .... $= \frac{Pab(a+2b)\sqrt{3a(a+2b)}}{27EIl}$

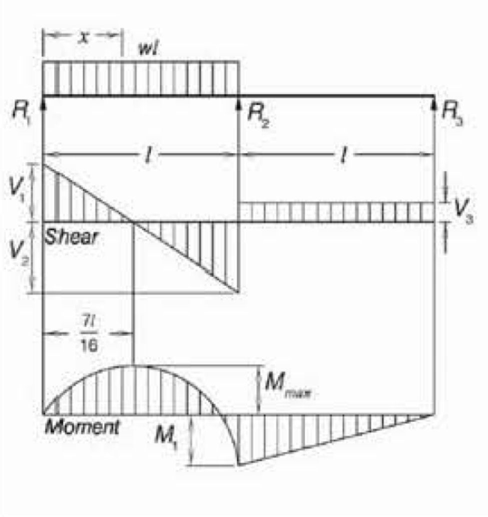
Δ_a (at point of load) .... $= \frac{Pa^2b^2}{3EIl}$

Δ_x (when $x < a$) .... $= \frac{Pbx}{6EIl}\left(l^2 - b^2 - x^2\right)$

Δ_x (when $x > a$) .... $= \frac{Pa(l-x)}{6EIl}\left(2lx - x^2 - a^2\right)$

Δ_{x_1} .... $= \frac{Pabx_1}{6EIl}(l + a)$

29. CONTINUOUS BEAM — TWO EQUAL SPANS — UNIFORM LOAD ON ONE SPAN



Total Equiv. Uniform Load .... $= \frac{49}{64} wl$

$R_1 = V_1$ .... $= \frac{7}{16} wl$

$R_2 = V_2 + V_3$ .... $= \frac{5}{8} wl$

$R_3 = V_3$ .... $= -\frac{1}{16} wl$

V_2 .... $= \frac{9}{16} wl$

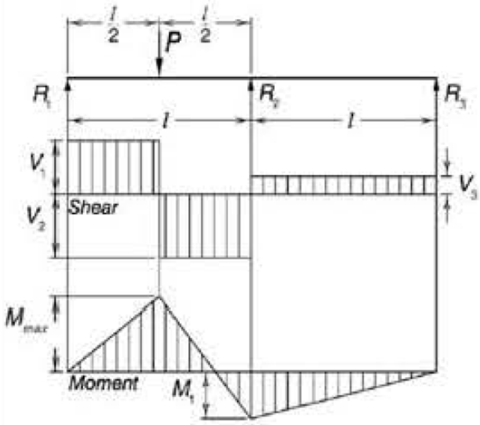
M_{max} (at $x = \frac{7}{16} l$) .... $= \frac{49}{512} wl^2$

M_1 (at support R_2) .... $= \frac{1}{16} wl^2$

M_x (when $x < l$) .... $= \frac{wx}{16}(7l - 8x)$

Δ_{max} (at 0.472 l from R_1)................... $= \frac{0.0092\, wl^4}{EI}$

30. CONTINUOUS BEAM — TWO EQUAL SPANS — CONCENTRATED LOAD AT CENTER OF ONE SPAN



Total Equiv. Uniform Load .... $= \frac{13}{8} P$

$R_1 = V_1$ .... $= \frac{13}{32} P$

$R_2 = V_2 + V_3$ .... $= \frac{11}{16} P$

$R_3 = V_3$ .... $= -\frac{3}{32} P$

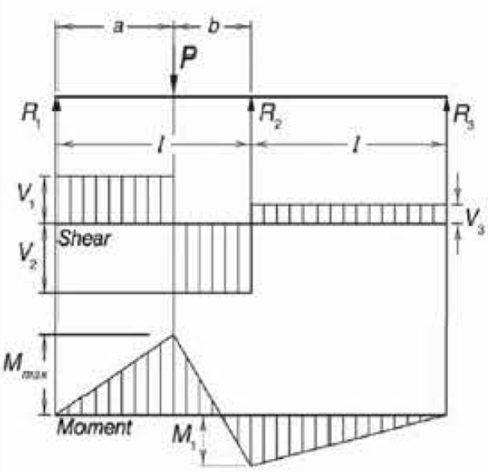
V_2 .... $= \frac{19}{32} P$

M_{max} (at point of load) .... $= \frac{13}{64} Pl$

M_1 (at support R_2) .... $= \frac{3}{32} Pl$

Δ_{max} (at 0.480 l from R_1) .... $= \frac{0.015\, Pl^3}{EI}$

31. CONTINUOUS BEAM — TWO EQUAL SPANS — CONCENTRATED LOAD AT ANY POINT



$R_1 = V_1$ .... $= \frac{Pb}{4l^3}\left(4l^2 - a(l + a)\right)$

$R_2 = V_2 + V_3$ .... $= \frac{Pa}{2l^3}\left(2l^2 + b(l + a)\right)$

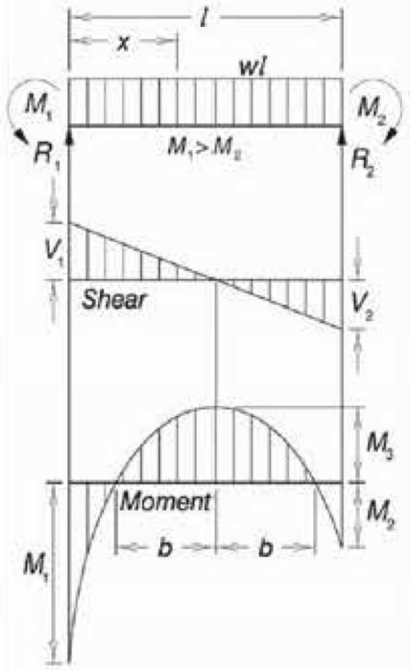
$R_3 = V_3$ .... $= -\frac{Pab}{4l^3}(l + a)$

V_2 .... $= \frac{Pa}{4l^3}\left(4l^2 + b(l + a)\right)$

M_{max} (at point of load) .... $= \frac{Pab}{4l^3}\left(4l^2 - a(l + a)\right)$

M_1 (at support R_2) .... $= \frac{Pab}{4l^2}(l + a)$

32. BEAM — UNIFORMLY DISTRIBUTED LOAD AND VARIABLE END MOMENTS



$R_1 = V_1$ .... $= \frac{wl}{2} + \frac{M_1 - M_2}{l}$

$R_2 = V_2$ .... $= \frac{wl}{2} - \frac{M_1 - M_2}{l}$

V_x .... $= w\left(\frac{l}{2} - x\right) + \frac{M_1 - M_2}{l}$

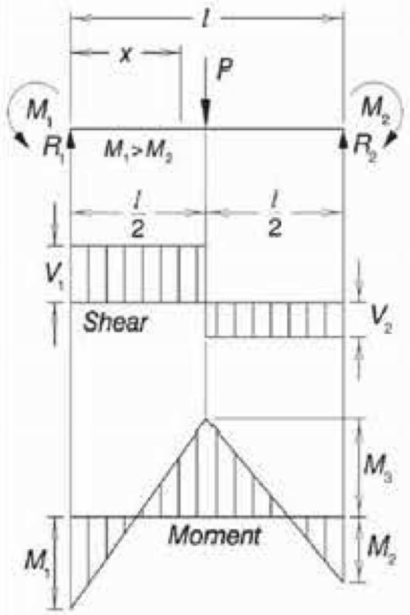
M_3 (at $x = \frac{l}{2} + \frac{M_1 - M_2}{wl}$) .... $= \frac{wl^2}{8} - \frac{M_1 + M_2}{2} + \frac{(M_1 - M_2)^2}{2wl^2}$

M_x .... $= \frac{wx}{2}(l - x) + \left(\frac{M_1 - M_2}{l}\right)x - M_1$

b (to locate inflection points) .... $= \sqrt{\frac{l^2}{4} - \left(\frac{M_1 + M_2}{w}\right) + \left(\frac{M_1 - M_2}{wl}\right)^2}$

$$\Delta_x = \frac{wx}{24EI}\left[x^3 - \left(2l + \frac{4M_1}{wl} - \frac{4M_2}{wl}\right)x^2 + \frac{12M_1}{w}x + l^3 - \frac{8M_1 l}{w} - \frac{4M_2 l}{w}\right]$$

33. BEAM — CONCENTRATED LOAD AT CENTER AND VARIABLE END MOMENTS



$R_1 = V_1$ .... $= \frac{P}{2} + \frac{M_1 - M_2}{l}$

$R_2 = V_2$ .... $= \frac{P}{2} - \frac{M_1 - M_2}{l}$

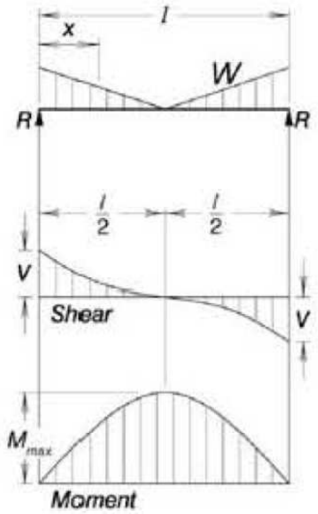
M_3 (at center) .... $= \frac{Pl}{4} - \frac{M_1 + M_2}{2}$

M_x (when $x < \frac{l}{2}$) .... $= \left(\frac{P}{2} + \frac{M_1 - M_2}{l}\right)x - M_1$

M_x (when $x > \frac{l}{2}$) .... $= \frac{P}{2}(l - x) + \frac{(M_1 - M_2)x}{l} - M_1$

Δ_x (when $x < \frac{l}{2}$) $= \frac{Px}{48EI}\left(3l^2 - 4x^2 - \frac{8(l - x)}{Pl}\left[M_1(2l - x) + M_2(l + x)\right]\right)$

34. SIMPLE BEAM — LOAD INCREASING UNIFORMLY FROM CENTER



Total Equiv. Uniform Load .... $= \frac{2W}{3}$

$R = V$ .... $= \frac{W}{2}$

V_x (when $x < \frac{l}{2}$) .... $= \frac{W}{2}\left(\frac{l-2x}{l}\right)^2$

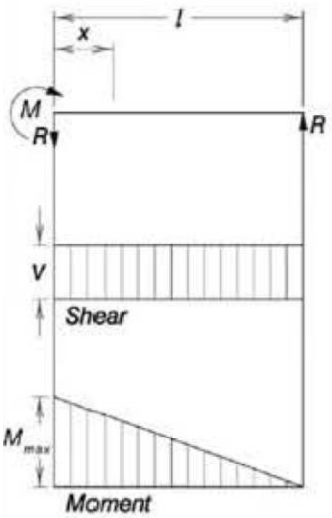
M_{max} (at center) .... $= \frac{Wl}{12}$

M_x (when $x < \frac{l}{2}$) .... $= \frac{W}{2}\left(x - \frac{2x^2}{l} + \frac{4x^3}{3l^2}\right)$

Δ_{max} (at center) .... $= \frac{3Wl^3}{320EI}$

Δ_x (when $x < \frac{l}{2}$) .... $= \frac{W}{12EI}\left(x^3 - \frac{x^4}{l} + \frac{2x^5}{5l^2} - \frac{3l^2x}{8}\right)$

35. SIMPLE BEAM — CONCENTRATED MOMENT AT END



Total Equiv. Uniform Load .... $= \frac{8M}{l}$

$R = V$ .... $= \frac{M}{l}$

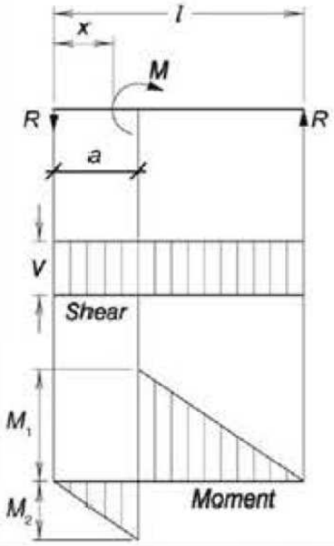
M_{max} .... $= M$

M_x .... $= M\left(1 - \frac{x}{l}\right)$

Δ_{max} (at $x = 0.423\ l$) .... $= 0.0642\,\frac{Ml^2}{EI}$

Δ_x .... $= \frac{M}{6EI}\left(3x^2 - \frac{x^3}{l} - 2lx\right)$

36. SIMPLE BEAM — CONCENTRATED MOMENT AT ANY POINT



Total Equiv. Uniform Load .... $= \frac{8M}{l}$

$R = V$ .... $= \frac{M}{l}$

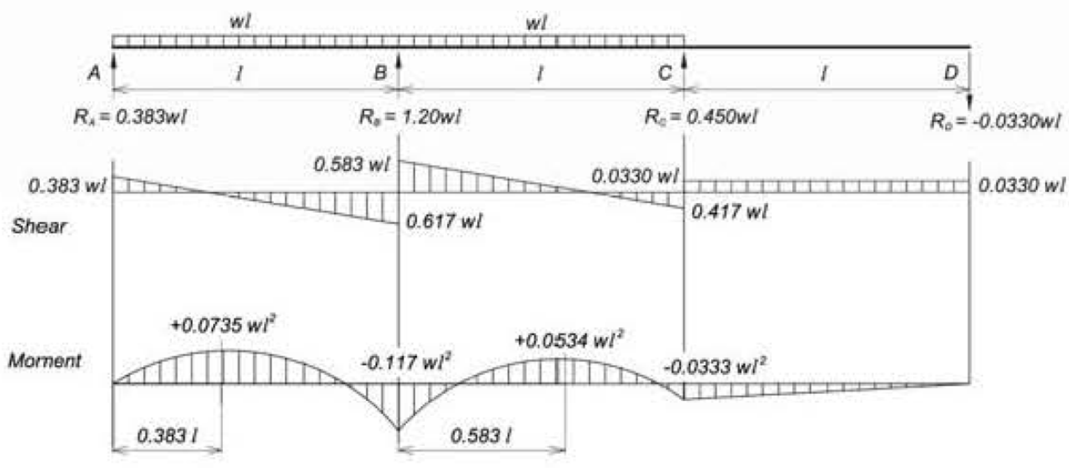
M_x (when $x < a$) .... $= Rx$

M_x (when $x > a$) .... $= R(l - x)$

Δ_x (when $x < a$) .... $= \frac{M}{6EI}\left[\left(6a - \frac{3a^2}{l} - 2l\right)x - \frac{x^3}{l}\right]$

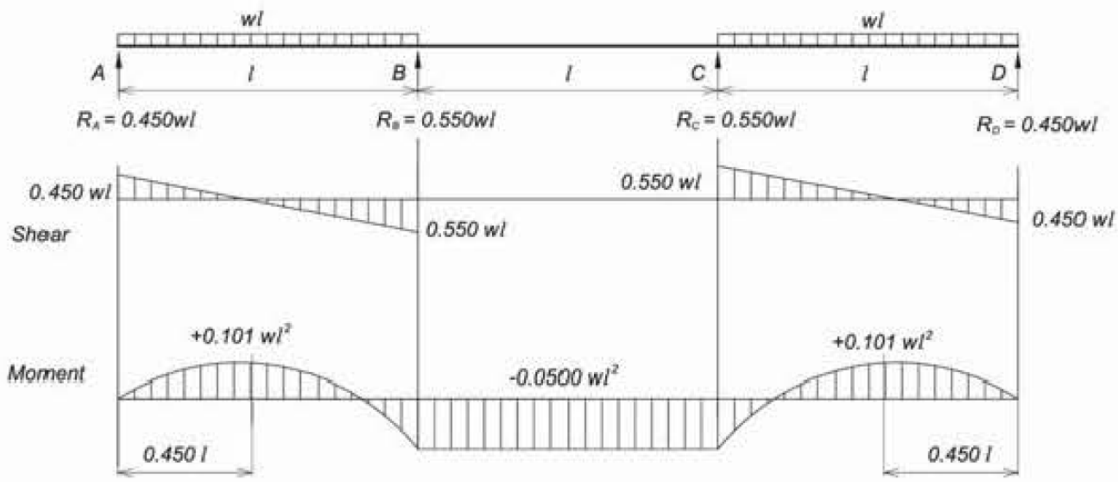
Δ_x (when $x > a$) .... $= \frac{M}{6EI}\left[3\left(a^2 + x^2\right) - \frac{x^3}{l} - \left(2l + \frac{3a^2}{l}\right)x\right]$

37. CONTINUOUS BEAM — THREE EQUAL SPANS — ONE END SPAN UNLOADED



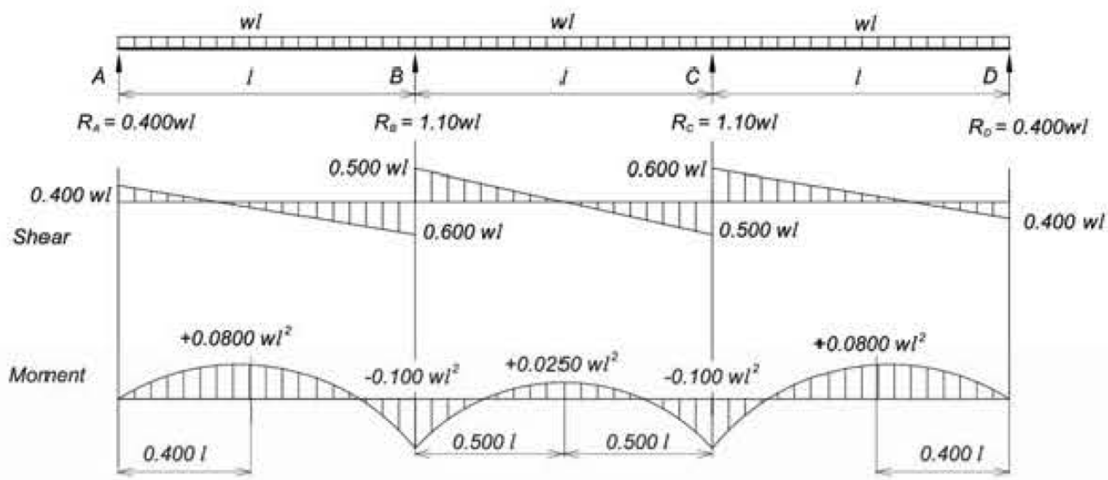
Δ_{max} (0.430l from A) = 0.0059 wl^4/EI

38. CONTINUOUS BEAM — THREE EQUAL SPANS — END SPANS LOADED



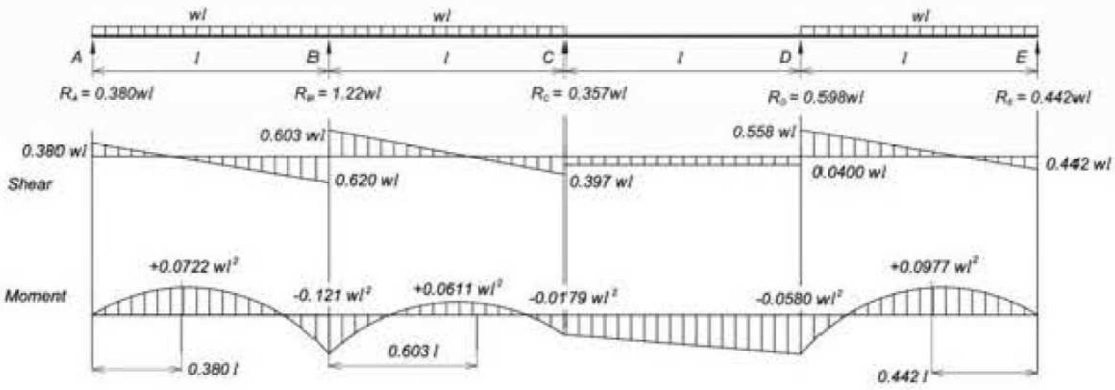
Δ_{max} (0.479l from A or D) = 0.0099 wl^4/EI

39. CONTINUOUS BEAM — THREE EQUAL SPANS — ALL SPANS LOADED



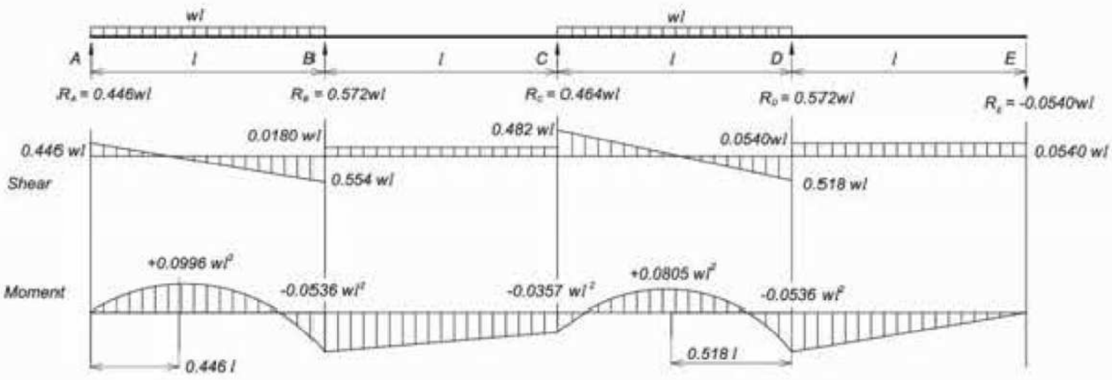
Δ_{max} (0.446l from A or D) = 0.0069 wl^4/EI

40. CONTINUOUS BEAM — FOUR EQUAL SPANS — THIRD SPAN UNLOADED



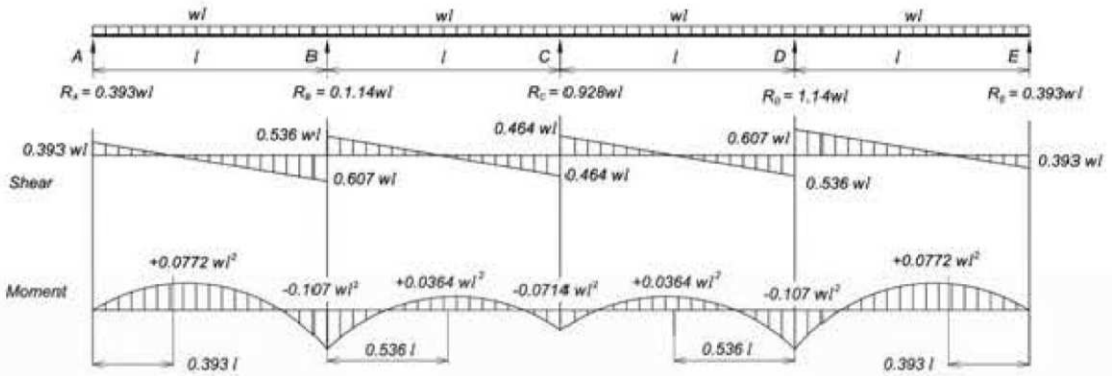
Δ_{max} (0.475 l from E) = 0.0094 wl^4/EI

41. CONTINUOUS BEAM — FOUR EQUAL SPANS — LOAD FIRT AND THIRD SPANS



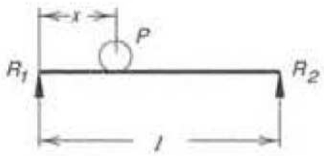
Δ_{max} (0.477 l from A) = 0.0097 wl^4/EI

42. CONTINUOUS BEAM — FOUR EQUAL SPANS — ALL SPANS LOADED



Δ_{max} (0.440 l from A and E) = 0.0065 wl^4/EI

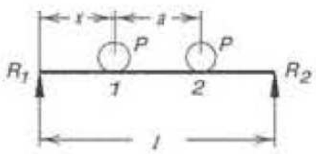
43. SIMPLE BEAM — ONE CONCENTRATED MOVING LOAD



$$R_{1\,max} = V_{1\,max}(\text{at } x = 0) \ldots\ldots = P$$

$$M_{max}\left(\text{at point of load, when } x = \frac{l}{2}\right) \ldots\ldots = \frac{Pl}{4}$$

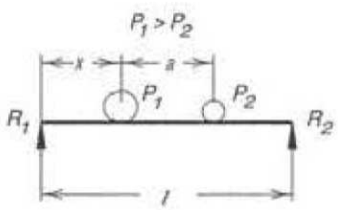
44. SIMPLE BEAM — TWO EQUAL CONCENTRATED MOVING LOADS



$$R_{1\,max} = V_{1\,max}(\text{at } x = 0) \ldots\ldots = P\left(2 - \frac{a}{l}\right)$$

$$M_{max}\begin{cases}\left[\begin{array}{l}\text{when } a < (2-\sqrt{2})l = 0.586l \\ \text{under load 1 at } x = \frac{1}{2}\left(l - \frac{a}{2}\right)\end{array}\right] \ldots\ldots = \frac{P}{2l}\left(l - \frac{a}{2}\right)^2 \\ \left[\begin{array}{l}\text{when } a > (2-\sqrt{2})l = 0.586l \\ \text{with one load at center of span (Case 43)}\end{array}\right] \ldots\ldots = \frac{Pl}{4}\end{cases}$$

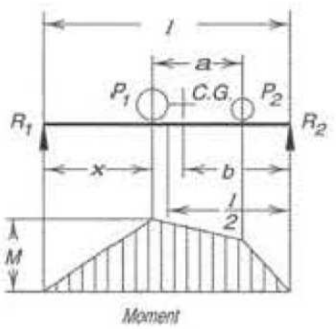
45. SIMPLE BEAM — TWO UNEQUAL CONCENTRATED MOVING LOADS



$$R_{1\,max} = V_{1\,max}(\text{at } x = 0) \ldots\ldots = P_1 + P_2\frac{l-a}{l}$$

$$M_{max}\begin{cases}\left[\text{under } P_1, \text{ at } x = \frac{1}{2}\left(l - \frac{P_2 a}{P_1 + P_2}\right)\right] \ldots\ldots = (P_1 + P_2)\frac{x^2}{l} \\ \left[\begin{array}{l}M_{max} \text{ may occur with larger} \\ \text{load at center of span and other} \\ \text{load off span (Case 43)}\end{array}\right] \ldots\ldots = \frac{P_1 l}{4}\end{cases}$$

GENERAL RULES FOR SIMPLE BEAMS CARRYING MOVING CONCENTRATED LOADS



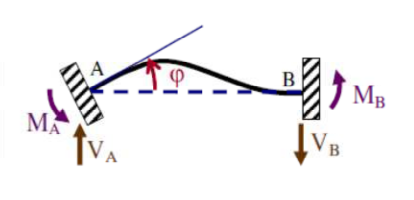
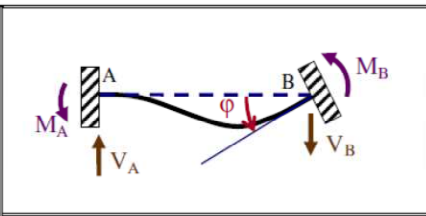
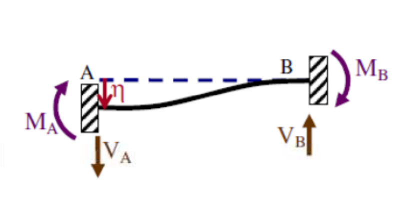
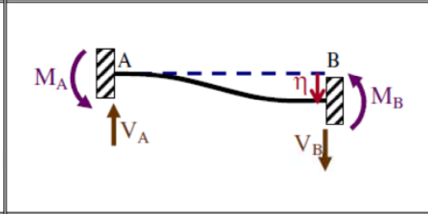
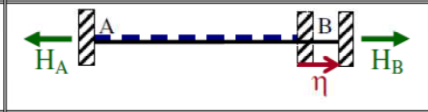
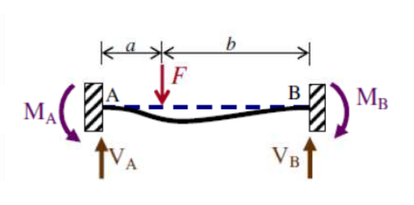
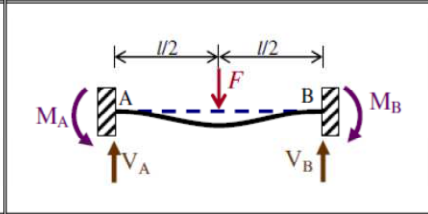
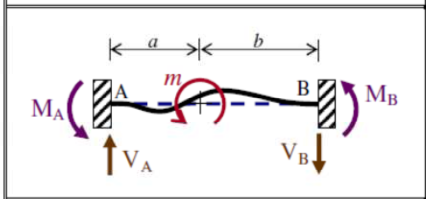
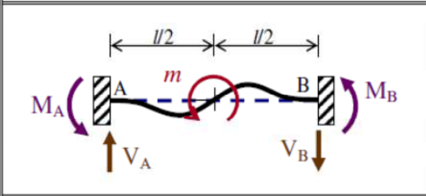
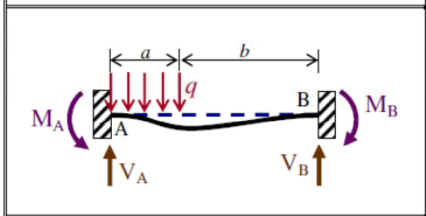
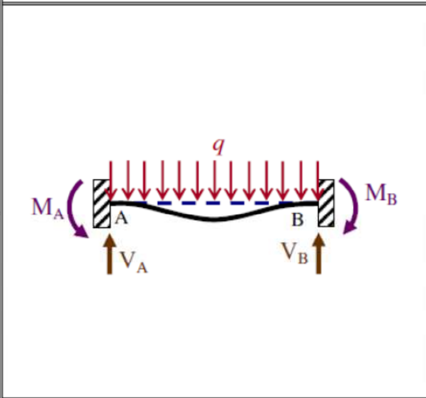
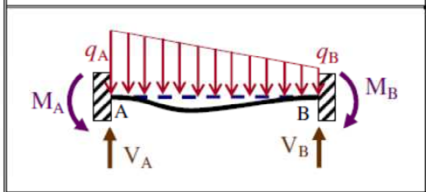
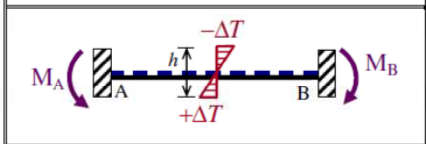
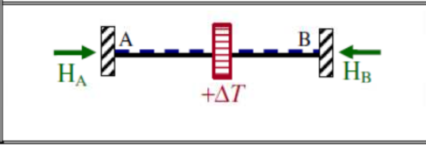
The maximum shear due to moving concentrated loads occurs at one support when one of the loads is at that support. With several moving loads, the location that will produce maximum shear must be determined by trial.

The maximum bending moment produced by moving concentrated loads occurs under one of the loads when that load is as far from one support as the center of gravity of all the moving loads on the beam is from the other support.

In the accompanying diagram, the maximum bending moment occurs under load P_1 when $x = b$. It should also be noted that this condition occurs when the center-line of the span is midway between the center of gravity of loads and the nearest concentrated load.

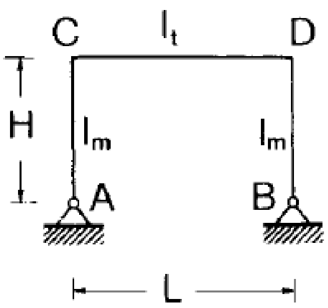
Reactions in end fixed beams

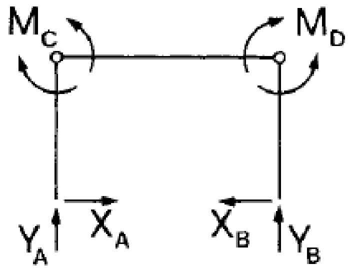
Note: l = beam length, h = transversal section height, A = area, I = inertia moment, E = Joung modulus , α = thermal dilatation coefficient. Positive values for the expression of the reactions corresponds to the sign assumed in the figure for the reactions.

<table>
<tr><td></td><td>$M_A = \frac{4EI}{l}\varphi$
$M_B = \frac{2EI}{l}\varphi$
$V_A = \frac{6EI}{l^2}\varphi$
$V_B = \frac{6EI}{l^2}\varphi$</td><td></td><td>$M_A = \frac{2EI}{l}\varphi$
$M_B = \frac{4EI}{l}\varphi$
$V_A = \frac{6EI}{l^2}\varphi$
$V_B = \frac{6EI}{l^2}\varphi$</td></tr>
<tr><td></td><td>$M_A = \frac{6EI}{l^2}\eta$
$M_B = \frac{6EI}{l^2}\eta$
$V_A = \frac{12EI}{l^3}\eta$
$V_B = \frac{12EI}{l^3}\eta$</td><td></td><td>$M_A = \frac{6EI}{l^2}\eta$
$M_B = \frac{6EI}{l^2}\eta$
$V_A = \frac{12EI}{l^3}\eta$
$V_B = \frac{12EI}{l^3}\eta$</td></tr>
<tr><td></td><td>$H_A = \frac{EA}{l}\eta$
$H_B = \frac{EA}{l}\eta$</td><td></td><td>$H_A = \frac{EA}{l}\eta$
$H_B = \frac{EA}{l}\eta$</td></tr>
<tr><td></td><td>$M_A = \frac{ab^2}{l^2}F$
$M_B = \frac{ba^2}{l^2}F$
$V_A = \frac{b^2}{l^3}(l+2a)F$
$V_B = \frac{b^2}{l^3}(l+2b)F$</td><td></td><td>$M_A = \frac{Fl}{8}$
$M_B = \frac{Fl}{8}$
$V_A = \frac{F}{2}$
$V_B = \frac{F}{2}$</td></tr>
<tr><td></td><td>$M_A = \frac{b}{l^2}(2a-b)m$
$M_B = \frac{a}{l^2}(2b-a)m$
$V_A = \frac{6ab}{l^3}m$
$V_B = \frac{6ab}{l^3}m$</td><td></td><td>$M_A = \frac{m}{4}$
$M_B = \frac{m}{4}$
$V_A = \frac{3}{2}\frac{m}{l}$
$V_B = \frac{3}{2}\frac{m}{l}$</td></tr>
<tr><td></td><td>$M_A = \frac{qa^2}{12l^2}(l^2+2lb+3b^2)$
$M_B = \frac{qa^3}{12l^2}(l+3b)$
$V_A = \frac{qa}{2l^3}[2l^3-a^2(l+b)]$
$V_B = \frac{qa^3}{2l^3}(l+b)$</td><td rowspan="2"></td><td rowspan="2">$M_A = \frac{ql^2}{12}$
$M_B = \frac{ql^2}{12}$
$V_A = \frac{ql}{2}$
$V_B = \frac{ql}{2}$</td></tr>
<tr><td></td><td>$M_A = \frac{l^2}{60}(3q_A+2q_B)$
$M_B = \frac{l^2}{60}(2q_A+3q_B)$
$V_A = \frac{l}{20}(7q_A+3q_B)$
$V_B = \frac{l}{20}(3q_A+7q_B)$</td></tr>
<tr><td></td><td>$M_A = 2EI\alpha\frac{\Delta T}{h}$
$M_B = 2EI\alpha\frac{\Delta T}{h}$</td><td></td><td>$H_A = EA\alpha\Delta T$
$H_B = EA\alpha\Delta T$</td></tr>
</table>

BEAM AND FRAME SCHEMES

a) SUPPORTED FRAME



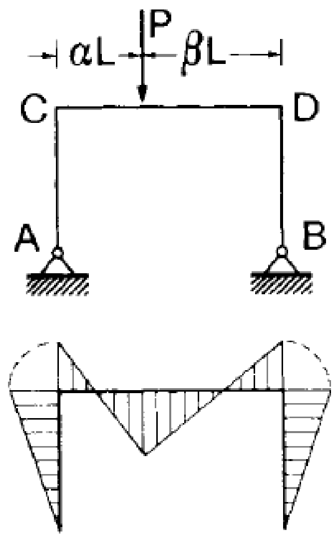


$$K_1 = \frac{H}{L}\frac{I_t}{I_m}$$

$$K_2 = 2\left(1 + \frac{2}{3}K_1\right)$$

Legenda

1.

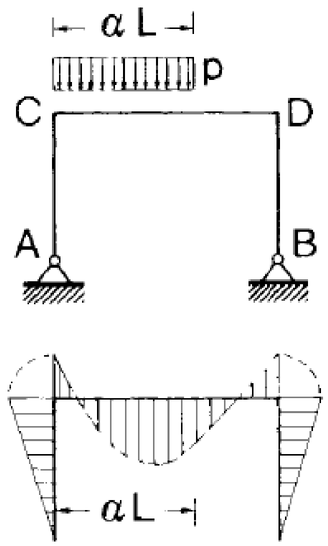


$$X_A = X_B = P\,\frac{L}{H}\,\frac{\alpha\,\beta}{K_2}$$

$$Y_A = P\,\beta \qquad Y_B = P\,\alpha$$

$$M_C = M_D = PL\,\frac{\alpha\,\beta}{K_2}$$

2.

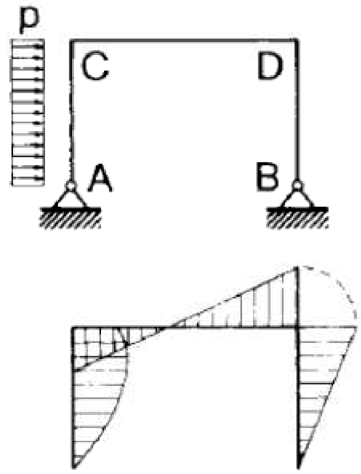


$$X_A = X_B = p\,\frac{\alpha^2 L^2}{6H}\,\frac{3-2\,\alpha}{K_2}$$

$$Y_A = p\,\alpha\,L\,\frac{2-\alpha}{2} \qquad Y_B = P\,\alpha^2\,\frac{L}{2}$$

$$M_C = M_D = p\,\frac{\alpha^2 L^2}{6}\,\frac{3-2\,\alpha}{K_2}$$

3.

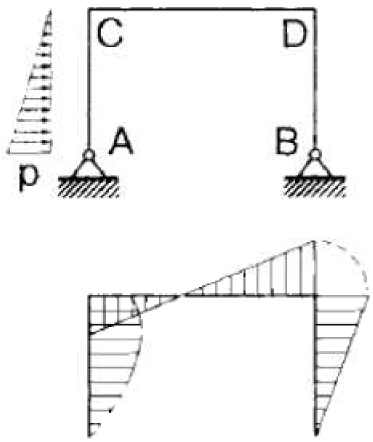


$$X_A = -p\,\frac{H}{4}\left(3-\frac{K_1}{3K_2}\right) \quad X_B = +p\,\frac{H}{4}\left(1+\frac{K_1}{3K_2}\right)$$

$$Y_A = -Y_B = -p\,\frac{H^2}{2L}$$

$$M_C = -p\,\frac{H^2}{4}\left(1-\frac{K_1}{3K_2}\right) \quad M_D = +p\,\frac{H^2}{4}\left(1+\frac{K_1}{3K_2}\right)$$

4.

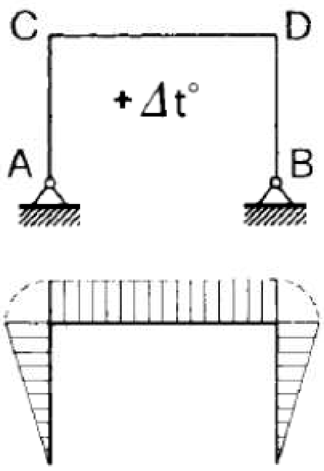


$$X_A = -p\,\frac{H}{180}\left(75-7\,\frac{K_1}{K_2}\right) \quad X_B = +p\,\frac{H}{180}\left(15+7\,\frac{K_1}{K_2}\right)$$

$$Y_A = -Y_B = -\frac{pH^2}{6L}$$

$$M_C = -p\,\frac{H^2}{180}\left(15-7\,\frac{K_1}{K_2}\right) \quad M_D = +p\,\frac{H^2}{180}\left(15+7\,\frac{K_1}{K_2}\right)$$

5.

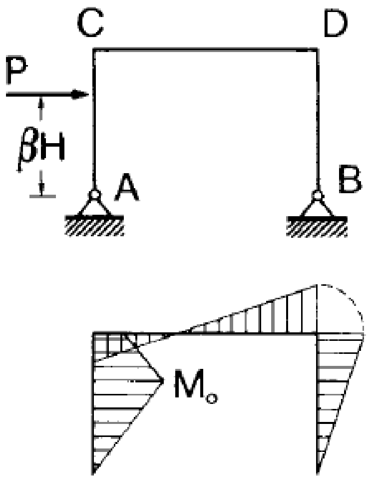


$$X_A = X_B = \frac{2\,E\,\alpha\,\Delta t\,I_t}{K_2\,H^2}$$

$$Y_A = Y_B = 0$$

$$M_C = M_D = \frac{2\,E\,\alpha\,\Delta t\,I_t}{K_2\,H}$$

6.



$$\chi = \frac{1 + K_1\ (1 - 1/3\ \beta^2)}{K^2}$$

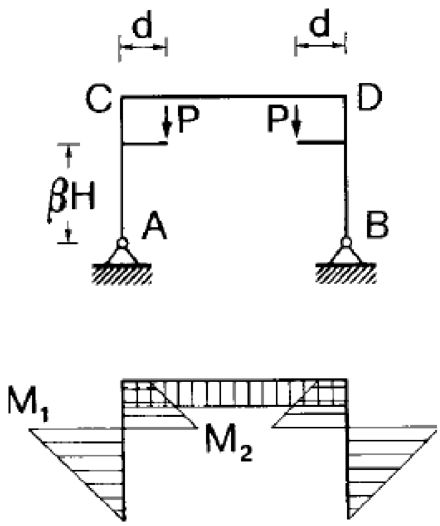
$$X_A = -P\ (1 - \beta\ \chi) \qquad X_B = P\ \beta\ \chi$$

$$Y_B = -Y_A = P\ \frac{H}{L}\ \beta$$

$$M_C = -P\ \beta\ H\ (1 - \chi) \qquad M_D = P\ \beta\ H\ \chi$$

$$M_0 = -X_A\ \beta\ H$$

7.

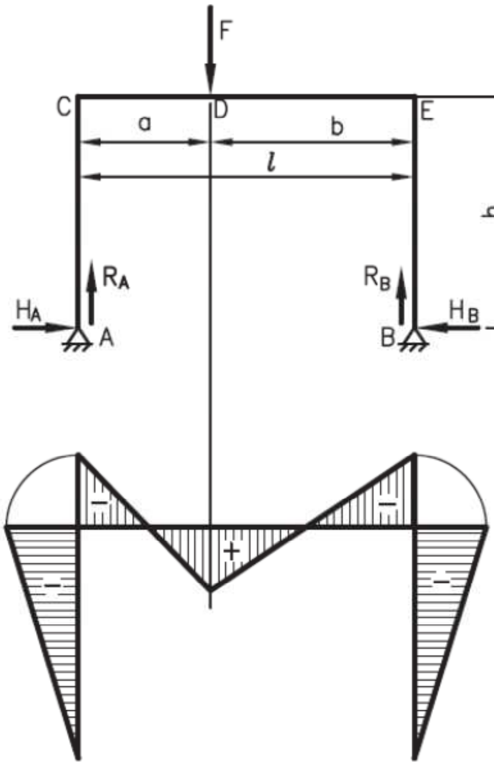


$$X_A = X_B = 2\ \frac{Pd}{H}\ \frac{1 + K_1\ (1 - \beta^2)}{K_2}$$

$$Y_A = Y_B = P$$

$$M_C = M_D = -Pd + X_A\ H$$

$$M_1 = X_A\ \beta\ H \qquad M_2 = Pd - X_A\ \beta\ H$$



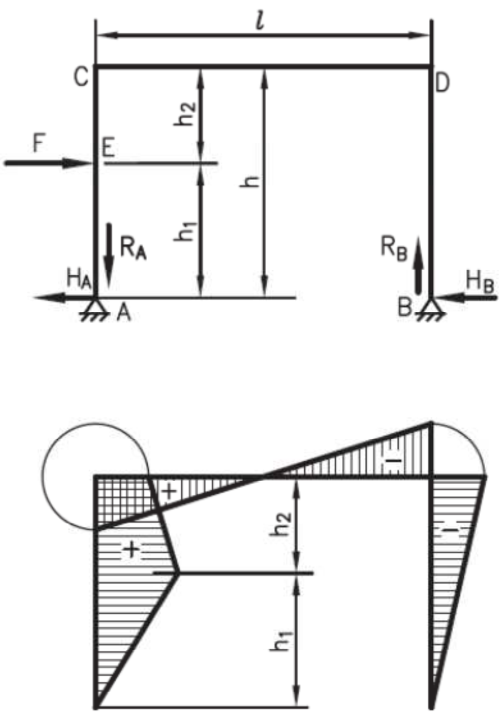
$$R_A = \frac{F \cdot b}{l}; \quad R_B = \frac{F \cdot a}{l}$$

$$k = \frac{h}{l}$$

$$H_A = H_B = \frac{3 \cdot F \cdot a \cdot b}{2 \cdot h \cdot l \cdot (2k+3)}$$

$$M_C = M_E = \frac{3 \cdot F \cdot a \cdot b}{2 \cdot l \cdot (2k+3)}$$

$$M_D = \frac{F \cdot a \cdot b}{2l} \cdot \frac{4k+3}{2k+3}$$



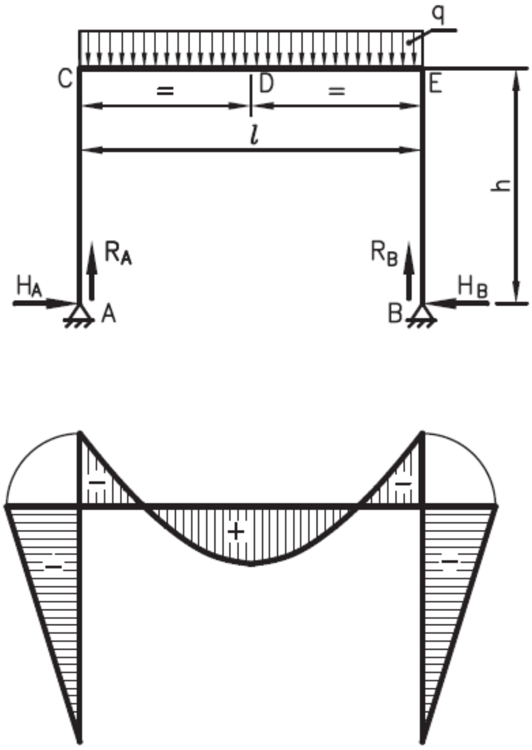
$$R_A = R_B = \frac{F \cdot h_1}{l}$$

$$H_A = \frac{F \cdot h}{2 \cdot l} \cdot \left[\frac{4h^3 + h_1^3 - 3h_1 \cdot h^2}{h \cdot (2h+3l)} + \frac{6h \cdot l - 3h_1 \cdot h \cdot l}{h \cdot (2h+3l)}\right]$$

$$H_B = F - H_A$$

$$M_E = H_A \cdot h; \quad M_C = H_A \cdot h - F \cdot h_2$$

$$M_D = -H_B \cdot h = -(F - H_A) \cdot h$$

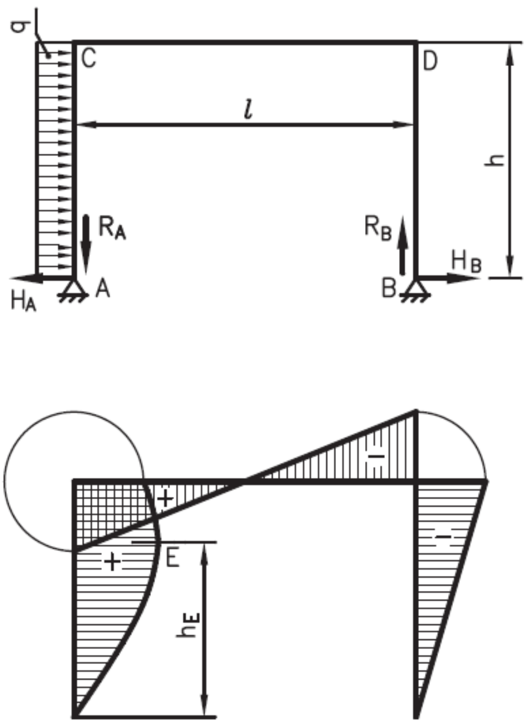


$$R_A = R_B = \frac{q \cdot l}{2}$$

$$H_A = H_B = \frac{q \cdot l^2}{4h \cdot \left(2\frac{h}{l} + 3\right)}$$

$$M_D = \frac{q \cdot l^2}{8} \cdot \frac{2h + l}{2h + 3l}$$

$$M_C = M_E = \frac{q \cdot l^2}{4 \cdot \left(2\frac{h}{l} + 3\right)}$$



$$R_A = R_B = \frac{q \cdot h^2}{2l}$$

$$H_A = \frac{q \cdot h}{8} \cdot \frac{11h + 18l}{2h + 3l}$$

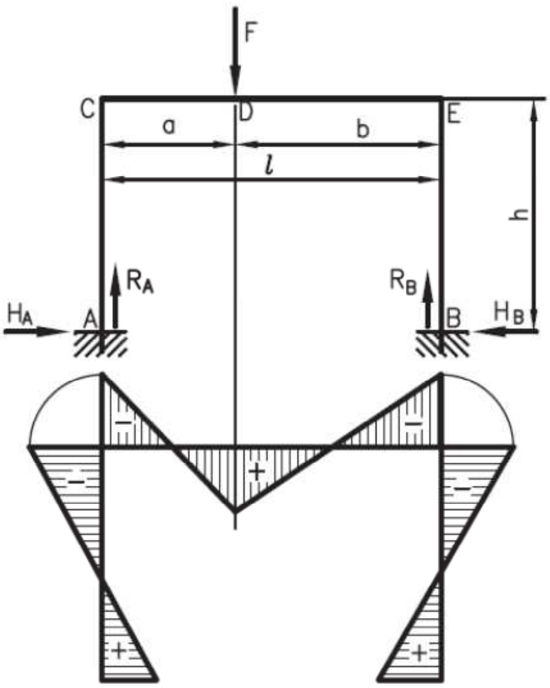
$$H_B = q \cdot h - H_A$$

$$M_E = \frac{q \cdot h^2}{128} \cdot \left(\frac{11k + 18}{2k + 3}\right)^2; \quad k = \frac{h}{l}$$

$$h_E = \frac{h}{8} \cdot \left(\frac{11k + 18}{2k + 3}\right)$$

$$M_C = \frac{3}{8} \cdot q \cdot l^2 \cdot \frac{h + 2l}{2h + 3l}$$

$$M_D = -\frac{q \cdot h^2}{8} \cdot \frac{5k + 6}{2k + 3}$$



$$R_A = \frac{F \cdot b}{l} \cdot \frac{6 \cdot h \cdot l + l^2 + a \cdot l - 2a^2}{6 \cdot h \cdot l + l^2}$$

$$R_B = \frac{F \cdot a}{l} \cdot \frac{6 \cdot h \cdot l + 3 \cdot a \cdot l - 2a^2}{6 \cdot h \cdot l + l^2}$$

$$H_A = H_B = \frac{3 \cdot F \cdot a \cdot b}{2 \cdot h \cdot l \cdot \left(\frac{h}{l} + 2\right)}$$

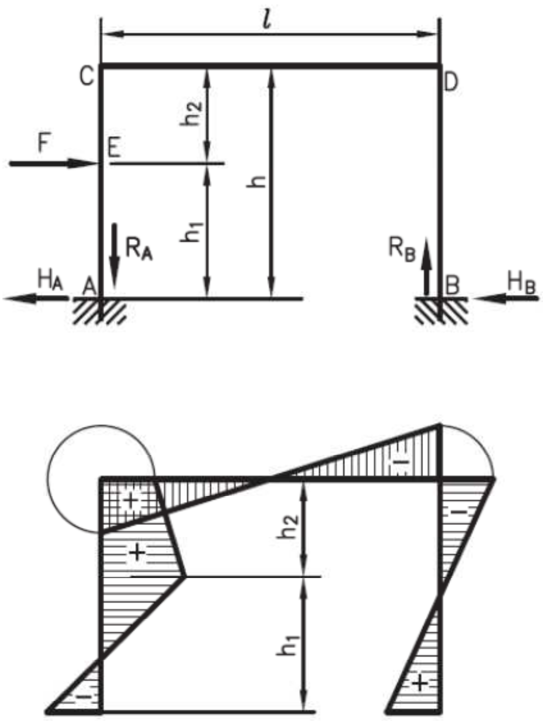
$$M_A = \frac{F \cdot a \cdot b}{2l} \cdot \frac{5 \cdot h \cdot l - l^2 + 2a \cdot (h + 2l)}{(h + 2l) \cdot (6h + l)}$$

$$M_B = \frac{F \cdot a \cdot b}{2l} \cdot \frac{3l + 7 \cdot h \cdot l - 2a \cdot (h + 2l)}{(h + 2l) \cdot (6h + l)}$$

$$M_C = M_A - H_A \cdot h$$

$$M_E = M_B - H_B \cdot h$$

$$M_D = M_A - H_A \cdot h + R_A \cdot a$$



$$R_A = R_B = \frac{3 \cdot F \cdot h_1^2}{6 \cdot h \cdot l + l^2}; \quad H_A = F - H_B$$

$$H_B = \frac{F \cdot h_1}{2 \cdot h^2 \cdot \left(\frac{h}{l} + 2\right)} \cdot \left[3 \cdot \left(\frac{h}{l} + 1\right) - \frac{h_1}{h} \cdot \right.$$

$$\left. \cdot \left(2 \cdot \frac{h}{l} + 1\right)\right]$$

$$M_A = -\frac{F \cdot h_1^2}{2h} \cdot \left[\frac{2h}{h_1} - \right.$$

$$\left. - \frac{3h \cdot l + 2h^2 - h \cdot (h + l)}{h^2 + 2 \cdot h \cdot l} - \frac{3h^2}{6 \cdot h^2 + h \cdot l}\right]$$

$$M_B = \frac{F \cdot h_1^2}{2 \cdot h} \cdot \left[\frac{3 \cdot h \cdot l + 2h^2 - h \cdot (h + l)}{h^2 + 2 \cdot h \cdot l} - \right.$$

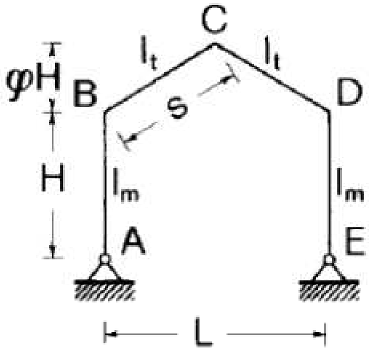
$$\left. - \frac{3h^2}{6 \cdot h^2 + h \cdot l}\right]$$

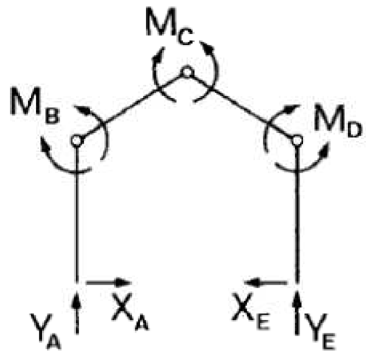
$$M_E = M_A + H_A \cdot h_1$$

$$M_D = M - H_B \cdot h$$

$$M_C = M_A + H_A \cdot h_1 - F \cdot h_2$$

b) SUPPORTED FRAME WITH SLOPED ROOF



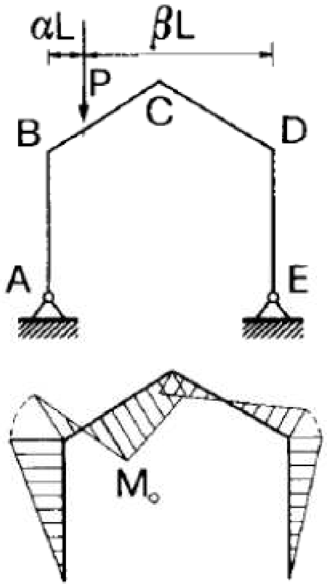


$$K_1 = \frac{H}{S}\,\frac{I_t}{I_m}$$

$$K_2 = (3 + K_1) + \varphi\,(3 + \varphi)$$

Legenda

1.



$$X_A = X_E = \frac{P\,\alpha\,L}{4H}\,\frac{1}{K_2}\,[6\beta + \varphi\,(3 - 4\,\alpha^2)]$$

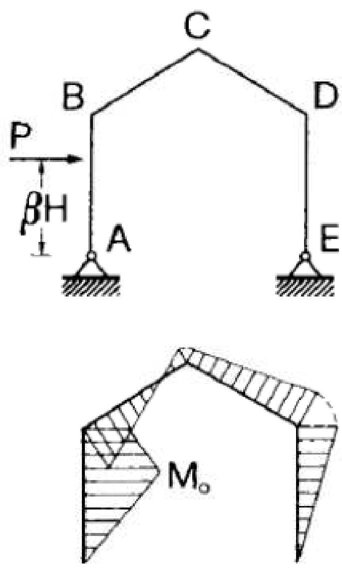
$$Y_A = P\,\beta \qquad Y_E = P\,\alpha$$

$$M_B = M_D = X_A\,H$$

$$M_C = -\frac{P\,\alpha\,L}{2} + X_A\,H\,(1 + \varphi)$$

$$M_0 = Y_A\,\alpha\,L - X_A\,H\,(1 + 2\,\varphi\,\alpha)$$

2.



$$X_E = \frac{P\,\beta}{4}\,\frac{1}{K_2}\,[K_1\,(3 - \beta^2) + 3\,(2 + \varphi)]$$

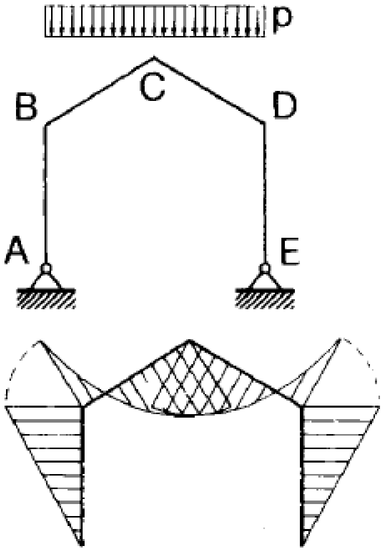
$$X_A = X_E - P$$

$$Y_E = -Y_A = P\,\beta\,H/L$$

$$M_B = X_E\,H - P\,\beta\,H \qquad M_C = X_E H\,(1 + \varphi) - P\,\beta\,\frac{H}{2}$$

$$M_D = X_E\,H \qquad M_0 = -X_A\,\beta\,H$$

3.



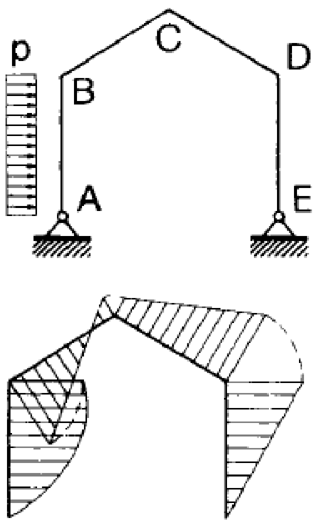
$$X_A = X_E = \frac{pL^2}{32H}\,\frac{1}{K_2}\,(8+5\,\varphi)$$

$$Y_A = Y_E = P\,\frac{L}{2}$$

$$M_B = M_D = X_A\,H$$

$$M_C = X_A\,H\,(1+\varphi) - p\,\frac{L^2}{8}$$

4.



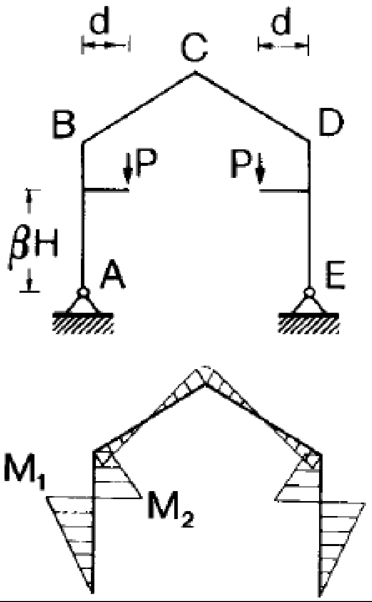
$$X_E = p\,\frac{H}{16}\,\frac{1}{K_2}\,[5\,K_1 + 6\,(1+\varphi)] \qquad X_A = X_E - pH$$

$$Y_E = -\,Y_A = p\,\frac{H^2}{2L}$$

$$M_B = X_E\,H - p\,\frac{H^2}{2} \qquad M_C = X_E\,H\,(1+\varphi) - p\,\frac{H^2}{4}$$

$$M_D = X_E\,H$$

5.



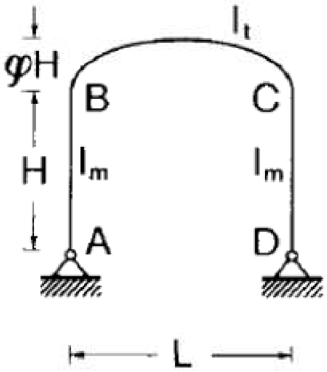
$$X_A = X_E = \frac{3}{2}\,\frac{Pd}{H}\,\frac{1}{K_2}\,[K_1\,(1-\beta^2) + (2+\varphi)]$$

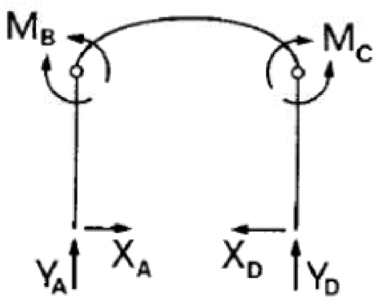
$$Y_A = Y_E = P$$

$$M_B = M_D = X_A\,H - Pd \qquad M_C = X_A\,H\,(1+\varphi) - Pd$$

$$M_1 = X_A\,\beta\,H \qquad M_2 = Pd - X_A\,\beta\,H$$

c) SUPPORTED FRAME WITH CURVED ROOF





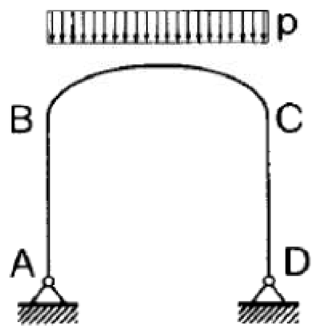
$$K_1 = \frac{H}{L}\,\frac{I_t}{I_m}$$

$$K_2 = 5\,(2K_1 + 3) + 4\varphi\,(5 + 2\varphi)$$

$$\varphi \leqslant 1$$

Legenda

1.

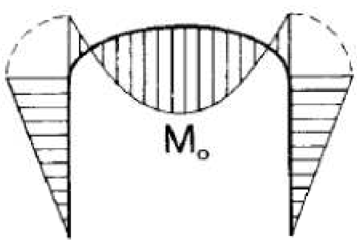


$$X_A = X_D = p\,\frac{L^2}{4H}\,\frac{1}{K_2}\,(5 + 4\,\varphi)$$

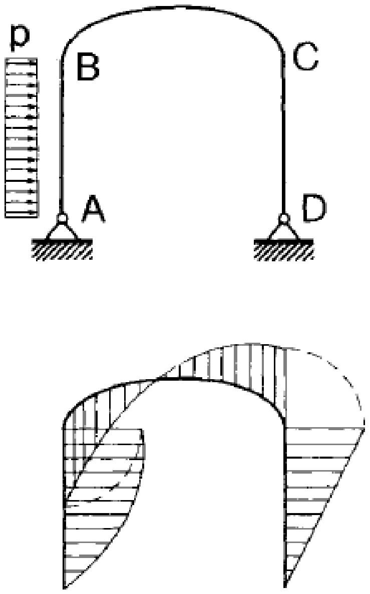
$$Y_A = Y_D = p\,\frac{L}{2}$$

$$M_B = M_C = X_A\,H$$

$$M_0 = p\,\frac{L^2}{8} - X_A\,H\,(1 + \varphi)$$



2.



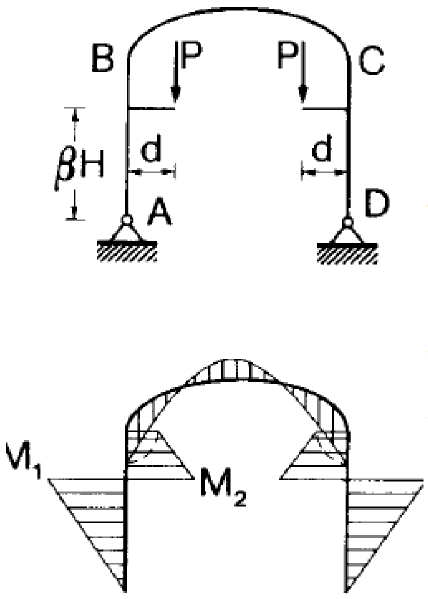
$$X_D = \frac{5}{8}\, pH\, \frac{1}{K_2}\, [(5\, K_1 + 6) + 4\, \varphi]$$

$$X_A = X_D - pH \qquad Y_A = Y_D = p\, \frac{H^2}{2L}$$

$$M_B = X_D\, H - p\, \frac{H^2}{2}$$

$$M_C = X_D\, H$$

3.



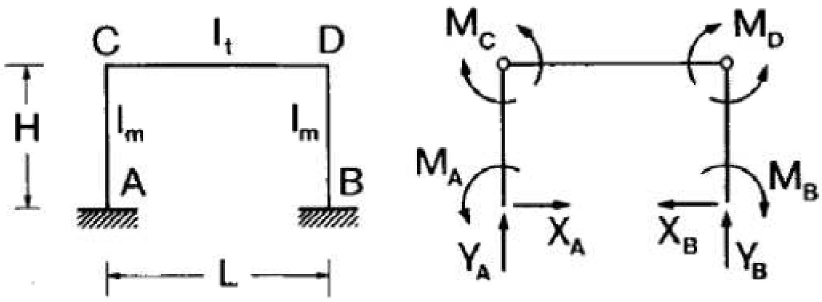
$$X_A = X_D = 5\, \frac{Pd}{H}\, \frac{1}{K^2}\, [3\, K_1\, (1 - \beta^2) + (3 + 2\, \varphi)]$$

$$Y_A = Y_D = P$$

$$M_B = M_C = X_A\, H - Pd$$

$$M_1 = X_A\, \beta\, H \qquad M_2 = Pd - X_A\, \beta\, H$$

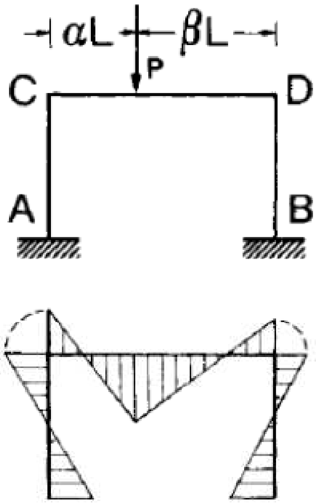
d) FIXED FRAME



$$K_1 = \frac{H}{L}\,\frac{I_t}{I_m}$$

Legenda

1.



$$\chi_1 = \frac{1}{2+K_1} \qquad \chi_2 = \frac{\beta-\alpha}{1+6K_1}$$

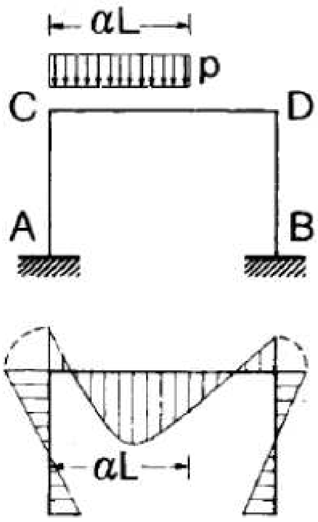
$$X_A = X_B = \frac{3}{2}\,P\,\frac{L}{H}\,\alpha\,\beta\,\chi_1$$

$$Y_A = P\,\beta\,(1+\alpha\,\chi_2) \qquad Y_B = P\,\alpha\,(1-\beta\,\chi_2)$$

$$M_A = -P\,\frac{L}{2}\,\alpha\,\beta\,(\chi_1-\chi_2) \qquad M_B = -P\,\frac{L}{2}\,\alpha\,\beta\,(\chi_1+\chi_2)$$

$$M_C = +P\,\frac{L}{2}\,\alpha\,\beta\,(2\chi_1+\chi_2) \qquad M_D = +P\,\frac{L}{2}\,\alpha\,\beta\,(2\chi_1-\chi_2)$$

2.



$$\chi_1 = \frac{3-2\alpha}{2+K_1} \qquad \chi_2 = \frac{(1-\alpha)^2}{1+6K_1}$$

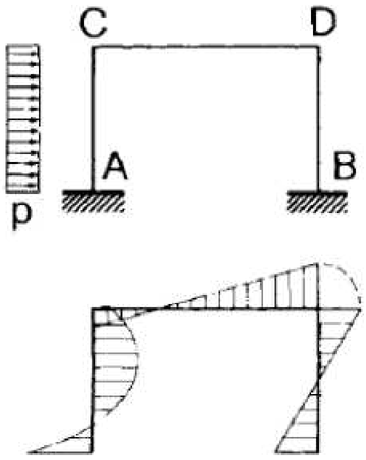
$$X_A = X_B = p\,\frac{\alpha^2\,L^2}{4H}\,\chi_1$$

$$Y_A = p\,\alpha\,L\left[1-\frac{\alpha}{2}\,(1-\chi_2)\right] \qquad Y_B = p\,\alpha^2\,\frac{L}{2}\,(1-\chi_2)$$

$$M_A = -p\,\frac{\alpha^2\,L^2}{12}\,(\chi_1-3\,\chi_2) \qquad M_B = -p\,\frac{\alpha^2\,L^2}{12}\,(\chi_1+3\,\chi_2)$$

$$M_C = +p\,\frac{\alpha^2\,L^2}{12}\,(2\chi_1+3\,\chi_2) \qquad M_D = +p\,\frac{\alpha^2\,L^2}{12}\,(2\chi_1-3\,\chi_2)$$

3.



$$\chi_1 = \frac{9+3K_1}{2+K_1} \quad \chi_2 = \frac{12K_1}{1+6K_1} \quad \chi_3 = \frac{K_1}{2+K_1}$$

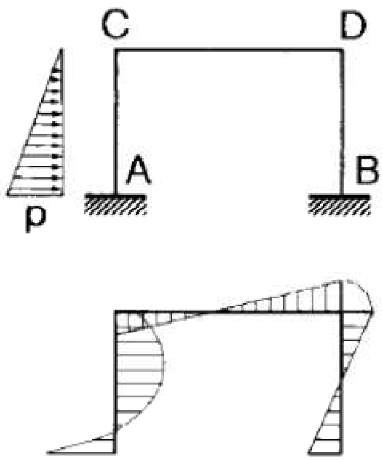
$$X_A = -p\,\frac{H}{8}\,\frac{13+6K_1}{2+K_1} \quad X_B = p\,\frac{H}{8}\,\frac{3+2K_1}{2+K_1}$$

$$Y_A = -Y_B = -p\,\frac{H^2}{L}\,\frac{K_1}{1+6K_1}$$

$$M_A = p\,\frac{H^2}{2} - p\,\frac{H^2}{24}\,(\chi_1+\chi_2) \quad M_B = -p\,\frac{H^2}{24}\,(\chi_1-\chi_2)$$

$$M_C = +p\,\frac{H^2}{24}\,(\chi_3-\chi_2) \quad M_D = +p\,\frac{H^2}{24}\,(\chi_3+\chi_2)$$

4.



$$\chi_1 = \frac{12+7K_1}{2+K_1} \quad \chi_2 = \frac{15K_1}{1+6K_1} \quad \chi_3 = \frac{2K_1}{2+K_1}$$

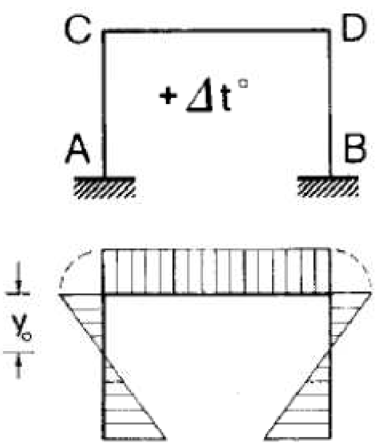
$$X_A = -p\,\frac{H}{40}\,\frac{36+17K_1}{2+K_1} \quad X_B = p\,\frac{H}{40}\,\frac{4+3K_1}{2+K_1}$$

$$Y_A = -Y_B = -p\,\frac{H^2}{4L}\,\frac{K_1}{1+6K_1}$$

$$M_A = p\,\frac{H^2}{6} - p\,\frac{H^2}{120}\,(\chi_1+\chi_2) \quad M_B = -p\,\frac{H^2}{120}\,(\chi_1-\chi_2)$$

$$M_C = +p\,\frac{H^2}{120}\,(\chi_3-\chi_2) \quad M_D = +p\,\frac{H^2}{120}\,(\chi_3+\chi_2)$$

5.



$$X_A = X_B = \frac{3E\,\alpha\,\Delta t\,I_t}{H_2}\,\frac{1+2K_1}{K_1\,(2+K_1)}$$

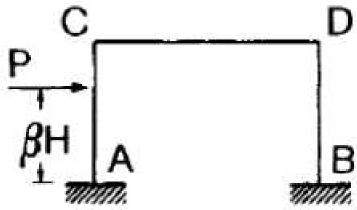
$$Y_A = Y_B = 0$$

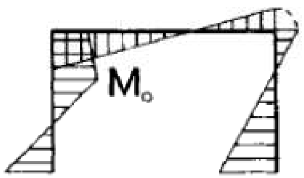
$$M_A = M_B = -\frac{3E\,\alpha\,\Delta t\,I_t}{H}\,\frac{1+K_1}{K_1\,(2+K_1)}$$

$$M_C = M_D = +\frac{3E\,\alpha\,\Delta t\,I_t}{H}\,\frac{1}{2+K_1}$$

$$y_0 = H\,\frac{K_1}{1+2K_1}$$

6.





$$\chi_1 = \frac{1 + (2-\beta)\,(1+K_1)}{2+K_1} \qquad \chi_2 = \frac{3K_1}{1+6K_1}$$

$$\chi_3 = \frac{(1-\beta)\,K_1}{2+K_1} \qquad \chi_4 = \frac{3\,(1+K_1) - \beta\,(1+2K_1)}{2+K_1}$$

$$X_A = -P + \frac{P}{2}\,\beta^2\,\chi_4 \qquad X_B = \frac{P}{2}\,\beta^2\,\chi_4$$

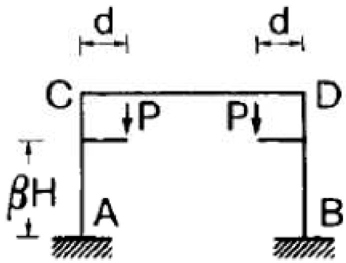
$$Y_A = -Y_B = -P\,\frac{H}{L}\,\beta^2\,\chi_2$$

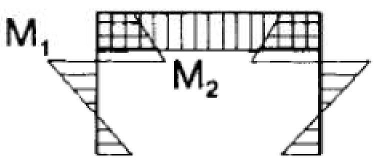
$$M_A = PH\,\beta - \frac{PH}{2}\,\beta^2\,(\chi_1 + \chi_2)$$

$$M_B = -\frac{PH}{2}\,\beta^2\,(\chi_1 - \chi_2) \qquad M_C = \frac{PH}{2}\,\beta^2\,(\chi_3 - \chi_2)$$

$$M_D = P\,\frac{H}{2}\,\beta^2\,(\chi_3 + \chi_2) \qquad M_0 = -M_A - X_A\,\beta\,H$$

7.





$$X_A = X_B = 3\,\frac{Pd}{H}\,\frac{2\beta\,K_1\,(1-\beta) + \beta\,(2-\beta)}{2+K_1}$$

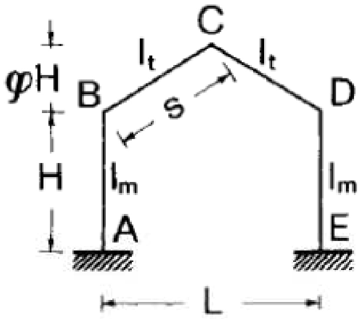
$$Y_A = Y_B = P$$

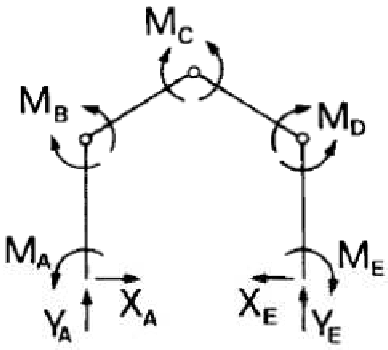
$$M_A = M_B = -Pd\,\frac{K_1\,(4\beta - 3\beta^2 - 1) + (6\beta - 3\beta^2 - 2)}{2+K_1}$$

$$M_C = M_D = M_A + X_A\,H - Pd$$

$$M_1 = M_A + X_A\,\beta\,H \qquad M_2 = -M_A - X_A\,\beta\,H + Pd$$

e) FIXED FRAME WITH SLOPED ROOF



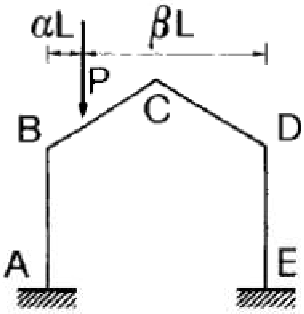


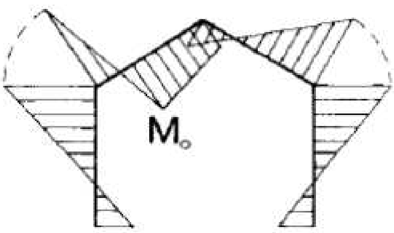
$$K_1 = \frac{H}{S}\,\frac{I_t}{I_m}$$

$$K_2 = (K_1+\varphi)^2 + 4\,K_1\,(\varphi^2+\varphi+1)$$

Legenda

1.





$$\chi_1 = \frac{1}{K_2}\,[2\beta\,K_1 + 3\varphi\,(2\alpha+K_1) - \varphi^2\,(1+4\alpha) +$$
$$- 4\alpha^2\,\varphi\,(2+K_1) - 4\,\alpha^2\,\varphi^2]$$

$$\chi_2 = \beta\,(\beta-\alpha)/(1+3K_1)$$

$$X_A = X_E = p\,\alpha\,\frac{L}{H}\,\frac{1}{K_2}\,[3K_1\,(1+\varphi) + -4\alpha^2\,\varphi\,(1+K_1)\ -3\alpha\,(K_1-\varphi)]$$

$$Y_A = P\,\beta + (M_A - M_E)/L \qquad Y_E = P\,\alpha - (M_A - M_E)/L$$

$$M_A = -\frac{P\,\alpha\,L}{2}\,(\chi_1-\chi_2) \qquad M_E = -\frac{P\,\alpha\,L}{2}\,(\chi_1+\chi_2)$$

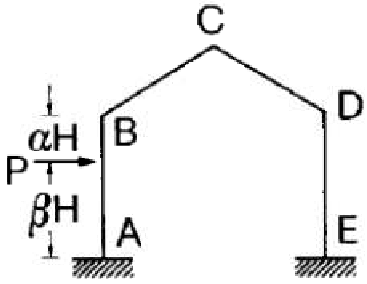
$$M_B = M_A + X_A\,H$$

$$M_C = M_E + X_A\,H\,(1+\varphi) - Y_E\,\frac{L}{2}$$

$$M_D = M_E + X_A\,H$$

$$M_0 = -M_A - X_A\,H\,(1+2\varphi\,\alpha) + Y_A\,\alpha\,L$$

2.



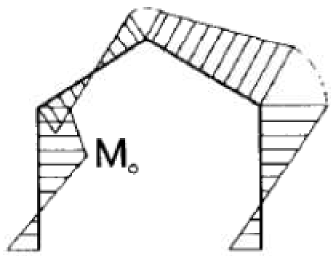
$$\chi_1 = \frac{1}{K_2}\,[K_1\,(4+K_1-2\beta\,K_1-6\beta+6\,\varphi)+\beta^2\,K_1\,(K_1+2+\varphi)+$$

$$+\,2\varphi\,K_1\,(2\varphi-\beta\varphi-3\beta)+\varphi^2]$$

$$\chi_2 = \frac{2+3K_1\,(2-\beta)}{2+6K_1}$$

$$X_E = \frac{P\,\beta}{2}\,\frac{1}{K_2}\,[6K_1\,(1+\varphi)+6\alpha\,\beta\,K_1^2+4\beta^2\,K_1\,(1+K_1)+$$

$$-\,3\beta\,K_1\,(2+K_1)-3\varphi\,K_1\,(1+\alpha)]$$



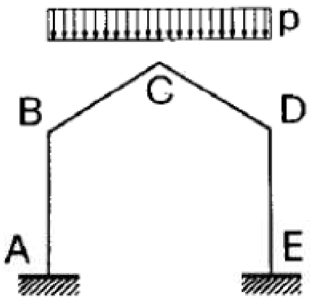
$$X_A = X_E - P \qquad Y_E = -\,Y_A = P\,\beta\,\frac{H}{L}-\frac{M_A-M_E}{L}$$

$$M_A = \frac{P\,\beta\,H}{2}\,(\chi_1+\chi_2) \qquad M_B = \frac{P\,\beta\,H}{2}\,(\chi_1-\chi_2)$$

$$M_C = M_E + X_E\,H\,(1+\varphi) - Y_E\,\frac{L}{2}$$

$$M_D = M_E + X_E\,H \qquad M_0 = -\,M_A - X_A\,\beta\,H$$

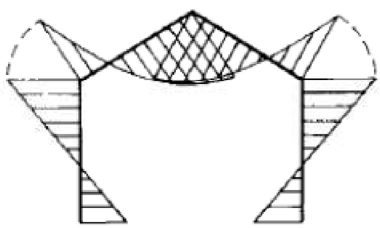
3.



$$X_A = X_E = P\,\frac{L^2}{8H}\,\frac{1}{K_2}\,[K_1\,(4+5\varphi)+\varphi]$$

$$Y_A = Y_E = P\,\frac{L}{2}$$

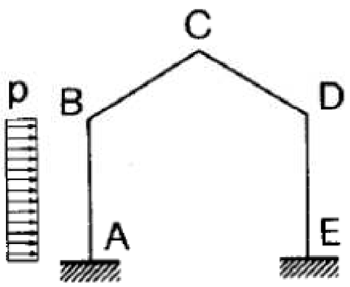
$$M_A = M_E = -\,P\,\frac{L^2}{48}\,\frac{1}{K_2}\,[K_1\,(8+15\varphi)+\varphi\,(6-\varphi)]$$



$$M_B = M_D = M_A + X_A\,H$$

$$M_C = -\,P\,\frac{L^2}{8} + M_A + X_A\,H\,(1+\varphi)$$

4.

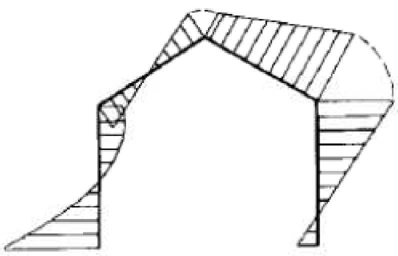


$$\chi_1 = \frac{1}{K_2}\,[K_1\,(6+K_1)+K_1\,\varphi\,(15+16\varphi)+6\,\varphi^2]$$

$$\chi_2 = (12K_1+6)/(1+3K_1)$$

$$X_E = p\,\frac{H}{4}\,\frac{1}{K_2}\,[K_1^2+K_1\,(3+2\varphi)] \qquad X_A = X_E - p\,H$$

$$Y_E = -Y_A = p\,\frac{H^2}{2L} - \frac{M_A - M_E}{L}$$

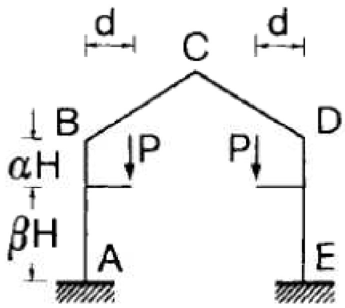


$$M_A = p\,\frac{H^2}{24}\,(\chi_1+\chi_2) \qquad M_E = p\,\frac{H^2}{24}\,(\chi_1-\chi_2)$$

$$M_B = M_A - p\,\frac{H^2}{2} + X_E\,H \qquad M_D = M_E + X_E\,H$$

$$M_C = M_E + X_E\,H\,(1+\varphi) - Y_E\,\frac{L}{2}$$

5.



$$X_A = X_E = 6\,\frac{Pd}{H}\,\frac{1}{K_2}\,[K_1\,(1+\varphi+\alpha)-\alpha\,(\alpha\,K_1+\alpha+\varphi)]$$

$$Y_A = Y_E = P$$

$$M_A = M_E = -Pd\,\frac{1}{K_2}\,[K_1\,(2\alpha\,K_1+2+3\varphi)-\alpha\,\varphi\,K_1\,\cdot$$

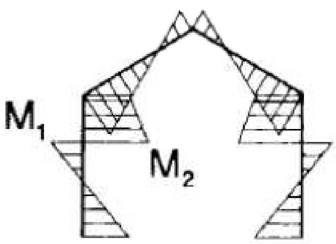
$$\cdot\,(6+3\,\alpha+4\varphi)-(3\alpha^2\,K_1^2+6\alpha^2\,K_1+\varphi^2)]$$

$$M_B = M_D = M_A - Pd + X_A\,H$$

$$M_C = M_A - Pd + X_A\,H\,(1+\varphi)$$

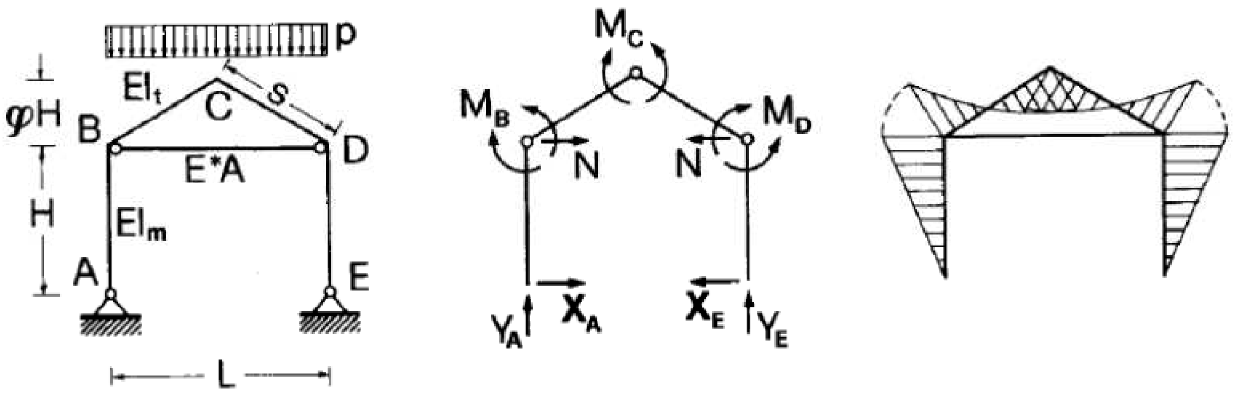
$$M_1 = M_A + X_A\,\beta\,H$$

$$M_2 = -M_A + Pd - X_A\,\beta\,H$$



f) SUPPORTED FRAME WITH TENSIONING HORIZONTAL BEAM

1.



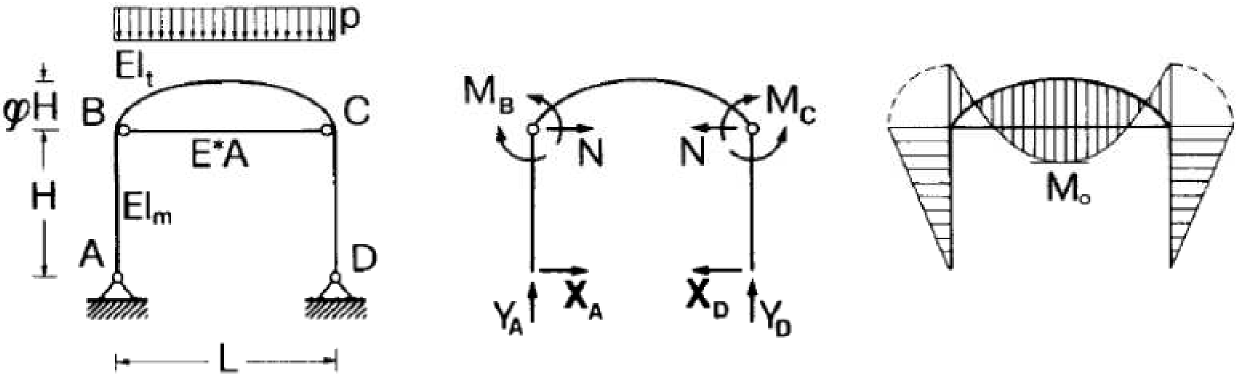
$$K_1 = \frac{H}{S}\,\frac{I_t}{I_m} \qquad K_2 = \left[1 + \frac{3}{2}\,\frac{E\,I_t}{E^*\,A}\,\frac{L}{\varphi^2 H_s^2}\right]^{-1} \qquad \lim_{E^*A \to \infty} K_2 = 1$$

$$X_A = X_E = p\,\frac{L^2}{16H}\,\frac{(16 - 15K_2) + 10\,\varphi\,(1 - K_2)}{(4K_1 + 12 - 9K_2) + 12\varphi\,(1 - K_2) + 4\varphi^2\,(1 - K_2)} \qquad Y_A = Y_E = p\,\frac{L}{2}$$

$$N = \frac{5}{32}\,p\,\frac{L^2}{\varphi\,H}\,K_2 - \left(\frac{3}{2}\,\frac{1}{\varphi} + 1\right) K_2\,X_A$$

$$M_B = M_D = X_A\,H \qquad M_C = -\,p\,\frac{L^2}{8} + X_A\,H\,(1 + \varphi) + N\,H\,\varphi$$

2.



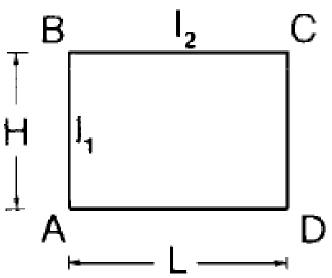
$$K_1 = \frac{H}{L}\,\frac{I_t}{I_m} \qquad K_2 = \frac{5 + 4\,\varphi}{5\,(3 + 2K_1) + 4\varphi\,(5 + 2\varphi)} \qquad K_3 = 1 - 2\,\varphi\,K_2$$

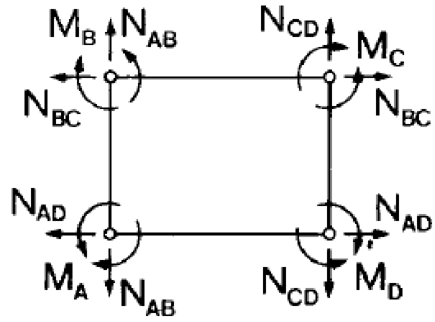
$$K_4 = \frac{E\,I_t}{E^*\,A}\,\frac{1}{\varphi^2\,H^2} \cdot \frac{1}{K_2^2\,(3 + 2K_1) - 2K_2\,K_3 + 0{,}4K_3^2} \qquad \lim_{E^*A \to \infty} K_4 = 0$$

$$X_A = X_D = p\,\frac{L^2}{4H}\,\frac{K_2 K_4}{1 + K_4} \qquad Y_A = Y_D = p\,\frac{L}{2} \qquad N = \frac{pL^2}{8H\,\varphi}\,\frac{1}{1 + K_4}$$

$$M_B = M_C = X_A\,H \qquad M_0 = p\,\frac{L^2}{8} - X_A\,H\,(1 + \varphi) - N\,H\,\varphi$$

g) CLOSED RECTANGULAR FRAME –

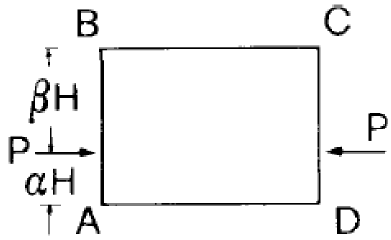


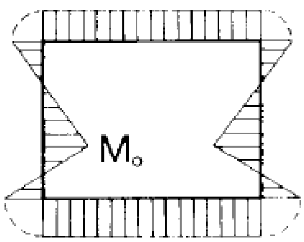


$$K_1 = \frac{H}{L}\,\frac{I_2}{I_1}$$

Legenda

1.





$$K_2 = (3+2K_1) + K_1\,(2+K_1)$$

$$M_B = M_C = \frac{PH\,\alpha\,\beta}{3}\,\frac{K_1}{K_2}\,[(1+\alpha)\,(3+2K_1) - K_1\,(1+\beta)]$$

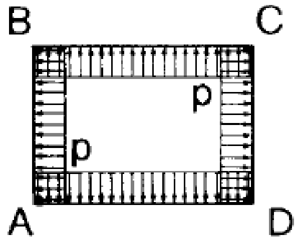
$$M_A = M_D = \frac{PH\,\alpha\,\beta}{3}\,\frac{K_1}{K_2}\,[(1+\beta)\,(3+2K_1) - K_1\,(1+\alpha)]$$

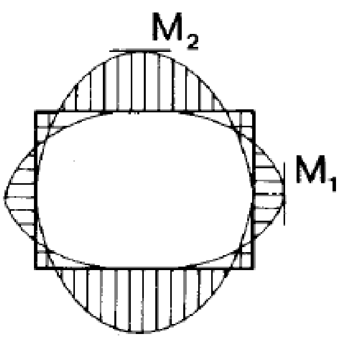
$$M_0 = PH\,\alpha\,\beta - M_A\,\beta - M_B\,\alpha$$

$$N_{AD} = -P\,\beta - \frac{M_A - M_B}{H} \qquad N_{AB} = N_{CD} = 0$$

$$N_{BC} = -P\,\alpha + \frac{M_A - M_B}{H}$$

2.





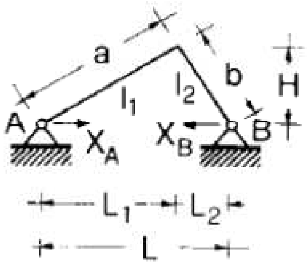
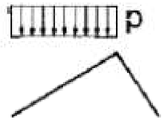
$$M_A = M_B = M_C = M_D = -\frac{p}{12}\,\frac{L^2 + H^2\,K_1}{1+K_1}$$

$$M_1 = p\,\frac{H^2}{8} + M_A \qquad M_2 = p\,\frac{L^2}{8} + M_A$$

$$N_{AB} = N_{CD} = p\,\frac{L}{2}$$

$$N_{AD} = N_{BC} = p\,\frac{H}{2}$$

h) SHED TYPE FRAME

 $K = \frac{a}{b}\frac{I_2}{I_1}$ Legenda		$X_A = X_B = \frac{pL_1^2}{8HL}\,\frac{4L_2 + K\,(L + 4L_2)}{1 + K}$
		$X_A = X_B - pH$ $X_B = \frac{pH}{8L}\,\frac{4L_2 + K\,(L_1 + 5L_2)}{1 + K}$